AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 14, NUMBER 4 JULY–AUGUST, 1946

The Pitch, Loudness and Quality of Musical Tones

A demonstration lecture introducing the new Tone Synthesizer*

HARVEY FLETCHER
Bell Telephone Laboratories, New York 14, New York

LET me begin by quoting from a textbook that is typical of most texts on general physics: "Pitch depends upon the *length* of the sound wave, loudness upon its *amplitude*, and quality or timbre upon the *form* of the complex wave."

The statement is true, but its implications are wrong, for one could say with equal verity that pitch depends upon the amplitude of vibration and also upon the *form* of the complex wave. Or one could say that loudness depends upon the form of the wave and also upon the length of the wave. It is interesting to note that by choosing properly the phases between the components of a tone it is possible to produce large changes in the wave form, and such changes produce no noticeable effects either in the quality, pitch or loudness of the tone. It is my purpose to discuss the relationships between the physical characteristics and the psychological characteristics of a musical tone.

The six physical aspects of a musical tone are its intensity, its fundamental frequency, its overtone structure, its duration, its growth and decay time pattern and its vibrato. We will be concerned with tones that are held steady except for the starting and stopping, and so vibrato will receive no consideration.

Let us review the meaning of each of these terms designating the physical aspects of the musical tone.

Intensity and Intensity Level

The intensity I in a free progressive wave is the acoustic power flowing through each unit area of the wave front. Acousticians have found it convenient to express the intensity level α in decibels. It is related to intensity by the equation

$$\alpha = 10 \log (I/I_0), \qquad (1)$$

where I_0, the reference intensity, has been assigned the value 10^{-16} w/cm^2 as an International Standard. The intensity level can be measured by sound level meters which are calibrated directly in decibels.

In the demonstration a microphone was hung above the audience in about the center of the hall. The scale of a sound level meter was projected on the screen. The scale read directly the intensity level in decibels near the microphone. It responded to the applause from the audience.

Frequency

Frequency f is a concept well known to physicists in all fields. It is measured in complete cycles per second.

* Given before the American Physical Society and the American Association of Physics Teachers, in New York, January 25, 1946. Also given as the fifteenth Joseph Henry Lecture of the Philosophical Society of Washington at the 1268th meeting of the Society in the U. S. National Museum, Washington, D. C., May 25, 1946.

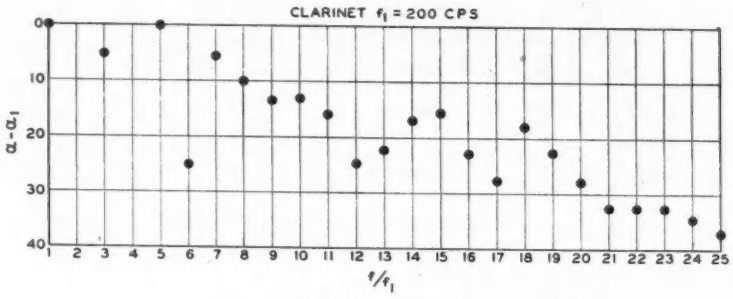


FIG. 1. Overtone structure of the clarinet; $f_1 = 200$ c/sec.

Overtone Structure

It is more difficult to give a concise definition of overtone structure. The data defining it are usually presented in the form shown in Table I. In the first line is given the ratio of the frequencies of the components. In the second line is given the number of decibels by which the intensity level of each component differs from that of the fundamental or of any reference level.

In Table II these data are shown more compactly. In the second line is shown the overtone structure of a complex tone having ten equally intense components. In the remaining three lines are shown the overtone structures for tones from a violin, a clarinet and a church bell. The overtone structure of the clarinet is also shown in the form of a graph in Fig. 1.

Such a graphical representation can be seen directly on the tone synthesizer shown in Fig. 2. The white knobs indicate the positions of the potentiometers for each of the overtones indicated. The vertical distances of the knobs from the top are proportional to the number of decibels that the overtone is attenuated.

Demonstration 1.—The synthesizer tone for the clarinet was sounded two or three times. Mr. Oncley then sounded the same tone on a clarinet so that the audience could judge how well it had been imitated.

It will be noted that no mention is made of the phase angle between components. As stated in the beginning, experimental tests have shown that changes in phase do not make perceptible changes in the auditory sensation except in rare cases. So in most acoustical work little attention is paid to phase angle.

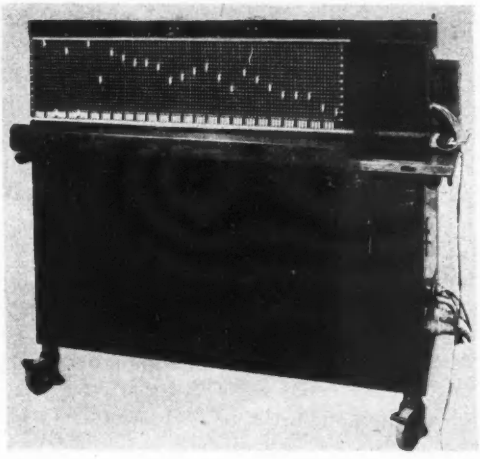


FIG. 2. Front view of the tone synthesizer.

Duration

In musical notation the duration is indicated by the kind of note to be played—that is, whether it is a whole, a half, a quarter or an eighth note. Physicists desire a more definite

TABLE I. Method of representing overtone structure.

Ratio of overtone to fundamental frequency	1	f_2/f_1	f_3/f_1	f_4/f_1	f_5/f_1	f_6/f_1	f_7/f_1	f_8/f_1
Intensity level difference (db)	$\alpha_0-\alpha_1$	$\alpha_0-\alpha_2$	$\alpha_0-\alpha_3$	$\alpha_0-\alpha_4$	$\alpha_0-\alpha_5$	$\alpha_0-\alpha_6$	$\alpha_0-\alpha_7$	$\alpha_0-\alpha_8$

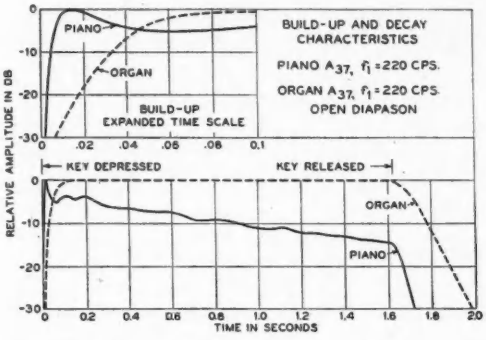


FIG. 3. Build-up and decay characteristics of an organ and a piano tone.

TABLE II. Compact method of representing various overtone structures.

Overtone structure representation S:

$S\rightarrow$	f_1	f_2/f_1	f_3/f_1	f_4/f_1	f_5/f_1	...	f_k/f_1
	0	$\alpha_2-\alpha_1$	$\alpha_3-\alpha_1$	$\alpha_4-\alpha_1$	$\alpha_5-\alpha_1$		$\alpha_k-\alpha_1$

Tone with ten equally intense components:

1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0

Violin tone:

1	2	3	4	5	6	7	8	9	10
0	1	20	9	15	5	15	20	14	14

Clarinet:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	∞	4	∞	0	25	6	10	13	13	16	25	22	17	16	22	28	18

19	20	21	22	23
22	28	32	32	35

Church bell:

3	7	11	17	19	24	29
35	7	3	0	5	13	27

concept, and they measure the variation in the intensity of the tone throughout its duration, from the attack to the release.

For example, Fig. 3 gives a time pattern for an organ tone and also one for a piano tone for a whole note. The ordinate gives the relative intensity levels and the abscissa the time. In the upper part of Fig. 3 there is an expanded time scale. Notice that the piano tone builds up to its maximum in about 1/100 sec, whereas it takes the organ tone ten times this long to come to its maximum.

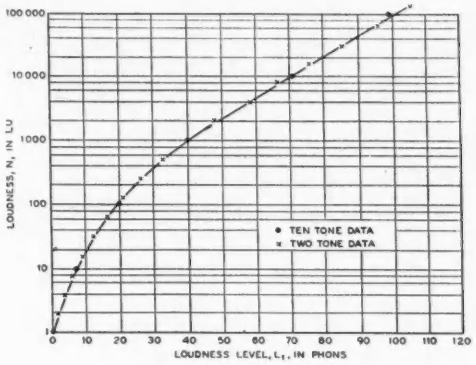


FIG. 4. Loudness scale for the reference tone.

Loudness

First let us see how the physical aspects of the tone affect the loudness. Loudness is the magnitude of the sensation. However, its quantitative use depends upon finding a method of measuring it and of describing a scale for representing it. Since we cannot put an instrument into the brain[1] and actually measure the magnitude of the nerve impulses reaching the auditory center, we must rely upon the judgment of individuals regarding their sensations. For this reason a reference tone has been adopted for making loudness comparisons. It is pure tone having a frequency of 1000 c/sec. For experimental purposes it is arranged so that any intensity level throughout the audible range can be produced.

A true loudness scale must be such that when the number of units on this scale is doubled, the magnitude of the sensation as experienced by typical observers will be doubled, or when trebled the loudness sensation is trebled, and so on. Also, if the two ears are equally sensitive, then when listening with both ears a sound should be twice as loud as when listening with one ear. Or if two equally loud tones are separated in frequency so that different sets of nerves are stimulated by each tone, then when they are sounding together the combination should be twice as loud as when each is sounding alone. Or if ten such tones are sounded together the

[1] Such a procedure is possible in experiments with animals.

combination should be ten times as loud as each component sounding alone. A scale that is found to satisfy all these conditions is shown in Fig. 4. The abscissas give the intensity level of the reference tone, and the ordinates the corresponding loudness in loudness units.

In the next demonstration the reference tone will be heard and one can note its intensity level by looking at the meter and then judge the loudness to see if it agrees with the curve.

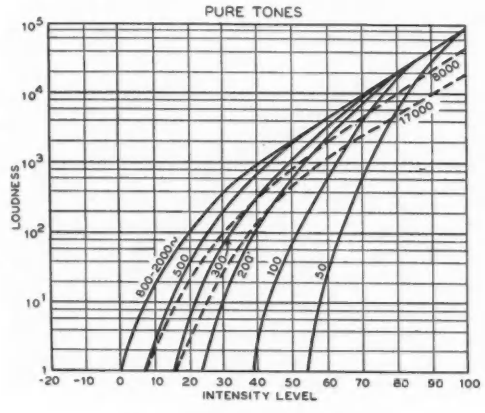


FIG. 5. Loudness of pure tones.

Remember that when loudness is doubled the intensity level is raised 9 db and the intensity increased eightfold. The intensity level will be raised in 9-db steps as will be seen by the meter, and the audience can judge if the loudness is doubled each step.

Demonstration 2.—The 1000-c/sec reference tone was sounded and carried from the threshold to the loudest levels available in the hall in sliding steps of 9 db each. The scale of the meter projected on the screen indicated that the levels picked up by a sound level meter increased in 9-db steps. Most of the audience judged that the loudness doubled each time.

In the upper part of Fig. 4 it will be noticed that if the loudness increases tenfold, the loudness level must be increased 30 db, or the intensity increases 1000-fold. In other words, in this region the so-called cube root law of loudness holds, namely,

$$N_1/N_2=(I_1/I_2)^{\frac{1}{3}}, \qquad (2)$$

where N_1 and N_2 are the two loudness values corresponding to the intensity values I_1 and I_2. This law holds not only for the reference tone but also approximately for all complex tones, including speech sounds. As a matter of fact, it was while trying to find a method for calculating the loudness loss of speech signals due to changes in a telephone system that this law was discovered.

If the overtones of a complex tone are separated in frequency so that they are sensed essentially by different sets of nerves, then the loudness values of the partial tones, listened to separately, can be added together to get the combined loudness when they are sounding together. In the next demonstration a tone having a fundamental frequency of 500 c/sec and ten equally loud components which are harmonics of 500 c/sec will be heard. The components are separated so that their individual loudness values may be added together to get the total loudness of the combined tone. Hence the combined ten components should be ten times louder than the single 1000-c/sec reference tone. In this demonstration the audience will act as observers in making such a loudness measurement on this ten-component tone.

Demonstration 3.—Each of the ten components from 500 to 5000 c/sec was sounded separately and its intensity adjusted so that the meter read approximately 60 db. Then all of them were sounded together, and the meter read 70 db. The meter was then turned off. First the audience was asked to compare the ten tones at 70 db with the 1000-c/sec reference tone at 95 db. Nearly all voted that the reference tone was the louder. When the 1000-c/sec tone was reduced to 90 db, the vote was divided. When it was reduced to 85 db, most of the audience voted that the ten tones were the louder. The point of balance was estimated to be 90 db. The meter was then turned on, and the audience was surprised to find that the 1000-c/sec reference tone read 90 db on the meter to balance the combined ten-tone reading of 70 db.

The curve of Fig. 4 then can be taken as defining our loudness scale for the reference tone. The loudness of any other tone, either pure or complex, can be determined by comparing it with this reference tone, adjusting the intensity of the latter until the two sound equally loud. The loudness of the adjusted reference tone on the foregoing scale is then the loudness of the unknown tone being measured.

In this way the loudness of a large number of

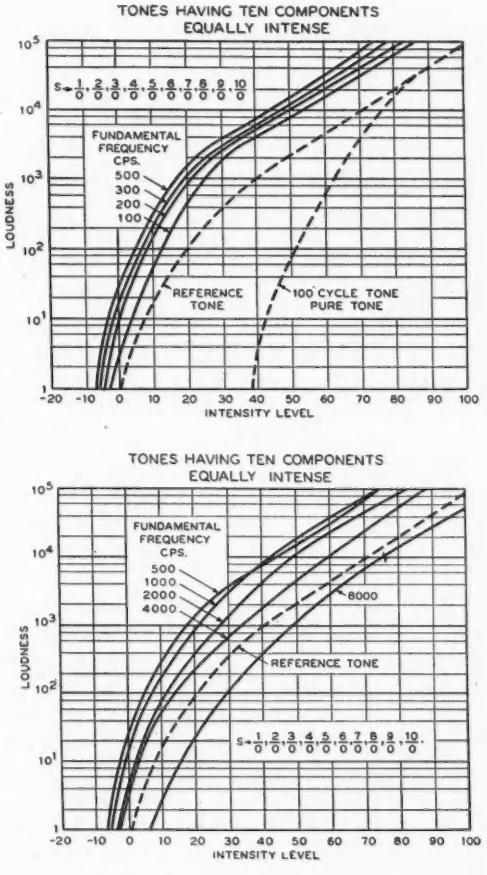
tones having various fundamental frequencies and overtone structures has been measured. In Fig. 5 the results of such measurements for pure tones are given. The abscissas give the intensity levels near the ear of the listener, and the ordinates give the loudness as judged by a number of observers.

It will be seen that tones with frequencies in the range from 800 to 2000 c/sec have approximately the same loudness when intensity levels are the same. There is only a small difference even for tones having frequencies as high as 5000 c/sec. For frequencies higher than this the loudness decreases rapidly as the frequency increases. It will be seen that for equal intensity levels the low frequencies are considerably less loud. For example, compare two pure tones of frequencies 100 and 1000 c/sec. When both have an intensity level of 40 db, the former has a loudness of 3 and the latter a loudness of 1000; however, for a common intensity level of 80 db they have approximately the same loudness, 20,000. This effect will now be demonstrated.

Demonstration 4.—A 100-c/sec tone was adjusted to sound equally loud to a 1000-c/sec tone when both were at a high intensity level (90 db). The attenuation before the loudspeaker was then gradually increased so that both tones were decreased 40 db. Before this point was reached the 100-c/sec tone was inaudible whereas the 1000-c/sec tone was still reasonably loud at the end.

It is this property of the low pitched tones that makes the bass in a musical selection much more prominent when it is heard at close range or reproduced very loudly than when heard at a distance or reproduced very softly.

Now let us see how the loudness is affected by adding an overtone structure to this 100-c/sec tone. Consider the overtone structure shown in Fig. 6. Each solid curve represents the relation between loudness and intensity level for a tone with *ten* components all equally intense. The dotted curves show the same relation for the reference tone and the 100-c/sec tone. The number attached to each curve gives the fundamental frequency. It is seen that changing the overtone structure from no overtones to nine equally intense ones has increased the loudness from 80 to 8000 for the particular tone having the fundamental frequency of 100 c/sec and an intensity level of 50 db. To illustrate this we will first listen to the 100-c/sec pure tone adjusted so that it is producing an intensity level of approximately 50 db near our ears. Next we will listen to a second 100-c/sec tone to which the first nine harmonics have been added which have the same intensity as the fundamental, and the intensity level of this complex tone adjusted to be also 50 db. In other words, it is arranged so that both the intensity and the frequency of the complex tone thus formed are the same as those of the pure 100-c/sec tone which we just heard. You will notice the very large increase in loudness. There is also a marked quality difference.



FIGS. 6 and 7. Loudness of tones having ten equally intense components.

Demonstration 5.—First the 100-c/sec tone was sounded at an intensity level of about 50 db. The harmonics were then added and the amplifier changed so as to decrease the intensity tenfold. The two tones were listened to alternately. While the meter indicated the same intensity level for each, the ten-component tone was markedly louder than the pure tone.

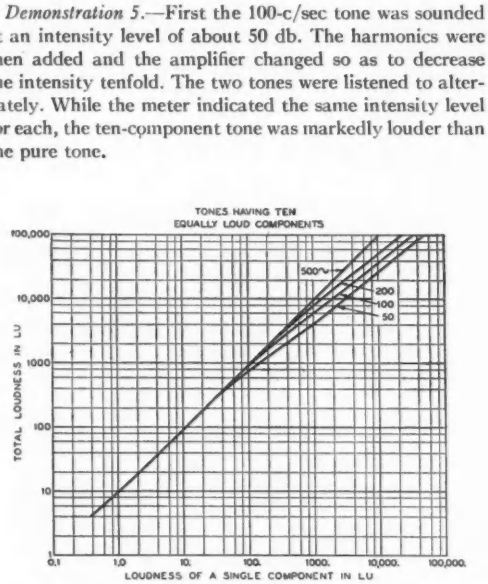


FIG. 8. Tones having ten equally loud components.

Let me emphasize that these two tones have the same intensity and, therefore, can be transmitted with the same electrical or mechanical power; yet the loudness of one is 80 and of the other is 8000. All tones are increased in loudness by making such a change in the overtone structure, but this increase is least at the higher intensities. It will be seen that for tones having such an overtone structure the loudest tones for a given intensity level are those having a fundamental frequency between 400 and 800 c/sec. These quantitative results show why it is easy to increase the loudness of a musical tone by increasing its overtone content, a practice that is common in producing musical tones. Practically all the loudness of the tones from the piano strings of low pitch is due to the higher overtones.

In Fig. 7 the results for the component tones with a fundamental frequency higher than 500 c/sec are given.

As stated previously, for a 500-c/sec tone having ten equally intense harmonics the components are sufficiently separated in frequency so that each one stimulates a separate patch of nerves on the end organ of hearing. Hence its loudness will be ten times that caused by each component when listened to separately. This was demonstrated in Experiment 3. However, as the frequency of separation becomes smaller these stimulated patches overlap and the total loudness from such a ten-component tone will always be less than ten times that produced by a single component acting alone. This is illustrated by the curves in Fig. 8.

These curves are for ten component tones, their intensity being adjusted so that when the components are listened to separately they sound equally loud. The numbers on the curves give the common difference in frequency between the components. For loudness values below about 600 lu, the combined loudness of the ten components is ten times that of each component. This is also true for higher loudness values if the frequency separation between the components is 500 c/sec or greater. For a frequency separation exceeding 1000 c/sec, it is difficult to make the higher components equally loud as the ear becomes insensitive for these very high frequencies.

When a group of components are close together in frequency and can be included in what is known as a critical band width, they can be treated as a single component having the combined intensity of the group. These critical band widths are approximately 60 c/sec for frequencies below 1000 c/sec and are equal to about 10 percent of the mid-frequency of the band for regions above 1000 c/sec.

In Fig. 9 there is a third set of curves which shows the relation between loudness and intensity level for four musical tones having the overtone structures noted in the figure. It is seen that the variation of loudness with intensity level is approximately the same for these musical tones as for the 1000-c/sec reference tone except at the higher intensities where their loudness is from 1.5 to 3 times greater than that of the reference tone.

We have seen how loudness depends upon frequency, intensity and overtone structure. Let us now consider briefly how it depends upon duration.

An electrically generated 1000-c/sec tone was transmitted to a heavily damped telephone receiver which was held to the ear of a listener.

The tone was then electrically interrupted so that its duration was varied from a few milliseconds to several seconds. Loudness measurements for each duration yielded the results shown in Fig. 10. The ordinates give the intensity level of the uninterrupted 1000-c/sec reference tone

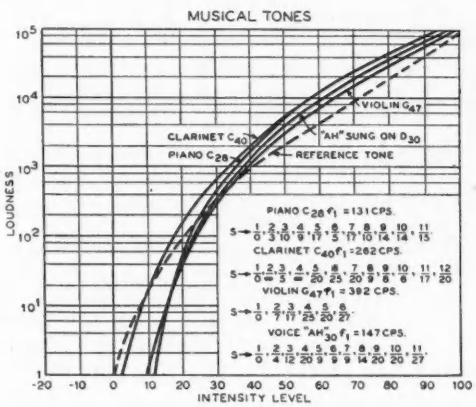


FIG. 9. Loudness of musical tones.

when it was adjusted to sound equally loud to the same tone interrupted at the rate indicated. For a constant intensity level the loudness increased with the duration until the latter reached about $\frac{1}{4}$ sec, after which it will be seen that the loudness is independent of the duration. Since most musical tones have durations greater than $\frac{1}{4}$ sec, it will be seen that loudness is independent of the duration for most tones that are used in music.

Enough has been given to show that the loudness of a musical tone as experienced by a typical observer depends upon the physical quantities intensity, frequency, overtone structure and duration. Nevertheless, intensity changes produce the major effect upon the loudness.

Pitch

The pitch of a musical tone is that characteristic of the auditory sensation which enables the listener to locate the position of the tone on a musical scale. It is probably related to the position of the stimulated nerve endings on the basilar membrane in the ear. Because of its subjective character the measurement of pitch can be made only by judgment tests. Since individuals vary in such tests, it is necessary to use several listeners and average the results in order to get something that will be characteristic of a "typical" listener.

As is the case for loudness, the pitch of a tone can be made definite only by choosing a reference standard. The reason why the need for such a reference tone has not been apparent in the past lies in the general acceptance of the false idea that the pitch is definitely determined by the frequency of vibration and that, consequently, a measurement of the latter determines the former. As we shall see, experiments indicate that this one-to-one relationship does not hold. Indeed, for complex tones composed of nonharmonic components the frequency of vibration is indeterminate. For most musical tones, however, the components have frequencies that are multiples of a fundamental frequency, and for such tones this fundamental frequency may be considered as the one corresponding to the tone. These are the reasons why it is necessary to define a reference tone and a scale for determining the pitch of a musical tone.

For this purpose a tone having a constant loudness of 1000 lu and capable of being varied in frequency throughout the audible range is used. We will arbitrarily define the pitch of our reference tone by its frequency of vibration. The pitch of any other tone, pure or complex, can be determined by comparing it with this chosen reference tone and by adjusting the latter until

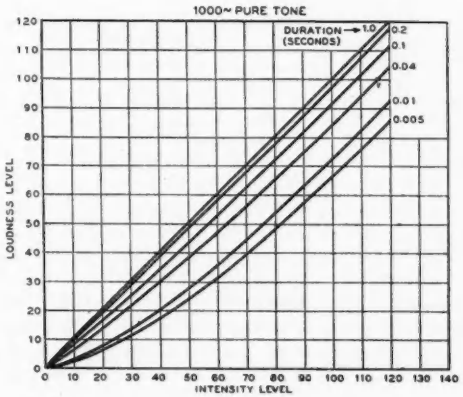


FIG. 10. Dependence of loudness on duration.

the two are judged to have the same pitch by a group of observers. For example, such a comparison shows that a tone having a vibration frequency of 200 c/sec and a loudness of 100,000 lu has a pitch of 180 c/sec—that is, its pitch drops

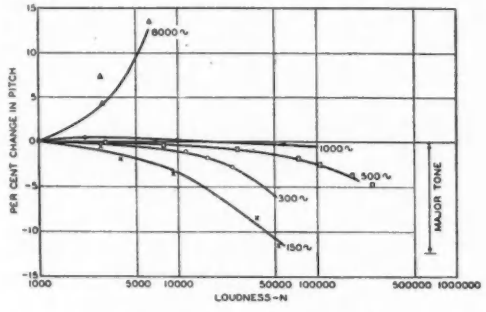


FIG. 11. Change in pitch as a function of loudness, for a pure tone. [Stevens].

20 c/sec as its loudness is increased from 1000 to 100,000 lu. Typical curves are shown in Fig. 11. The ordinates give the percentage change in pitch when the loudness is changed from 1000 lu to the value shown by the abscissa. The double arrows show the magnitude of a major tone on this scale. It will be seen that for large changes in loudness the pitch may change as much as a full musical tone without any change in frequency.

Although the data obtained by different observers for tones having frequencies below 100 c/sec are not very consistent, they indicate that the pitch change becomes smaller as the frequency is lowered. The pitch changes for tones having frequencies above 1000 c/sec are small, and for very high frequencies the pitch rises with increases in the intensity.

Demonstration 6.—To illustrate this change in pitch with increase in intensity we will try a demonstration that is somewhat difficult because the standing waves produced in the room make the intensity at some positions very much greater than at others. When the 150-c/sec tone is sounded the audience is asked first to listen with both ears; and then to close both ear canals as tightly as possible either by means of the fingers or by clasping the hands tightly over the ears. In so doing the intensity level in the ear will be decreased about 40 db and the change in pitch will be noted.

The following comparison is interesting. Let four tones, *A*, *B*, *C*, *D*, be defined as follows: *A*, frequency, 150 c/sec, and loudness, 1000 lu; *B*, frequency, 300 c/sec, and loudness, 1000 lu; *C*, frequency, 168 c/sec, and loudness, 50,000 lu; *D*, frequency, 318 c/sec, and loudness, 50,000 lu. Then it will be seen from Fig. 11 that tone *C* has the same pitch as *A*, and tone *D* the same pitch as *B*. Since *A* and *B* are just one octave apart in pitch, then *C* and *D* must be one octave apart in pitch although their frequencies are 168 and 318 c/sec, respectively. To check this result, *C* and *D* were sounded alternately and were judged to be just one octave apart in pitch. When they were sounded together, however, they were very discordant.

One now inquires what will happen to the pitch of a tone having ten components if some of the components in the lower pitch range are eliminated. The answer is that the pitch does not change. This remarkable fact was pointed out about 20 years ago, but the experiment showing this effect is rather striking, so I will repeat it here.

Demonstration 7.—(*a*) Components having 600, 700, 800, 900 and 1000 c/sec were sounded together. The pitch corresponded to 100 c/sec as judged by an observer from the audience.

(*b*) The components 400, 500, 600, 700, 800, 900 and 1000 c/sec were sounded and the pitch noted. Then the odd harmonics were eliminated and the combination sounded again. The pitch rose to 200 c/sec—that is, it rose an octave.

(*c*) Ten tones from 2000 to 2500 c/sec were sounded. To the surprise of the audience, the 50-c/sec subjective tone was louder than any component.

It should be emphasized that when the tones corresponding to frequencies 400, 600, 800 and 1000 c/sec are sounded together, the pitch will be found to be 200 c/sec. When to this combination, the frequencies of 500, 700 and 900 c/sec are added, the pitch of the combination drops to 100 c/sec. This experiment illustrates the peculiar phenomen that, when three components all of which have frequencies above 500 c/sec are added to a certain complex tone, the pitch of the latter is reduced from 200 to 100 c/sec.

A complex tone whose components have frequencies forming a harmonic series has a very

definite pitch. This pitch corresponds approximately but not exactly to that of the fundamental of the series. For example, as shown in Fig. 12, a tone having five such components and a fundamental frequency of 100 c/sec dropped in pitch only 2 percent when the loudness was raised from 1000 to 60,000 lu. This corresponds approximately to the change in pitch of either one of the upper three components if it were sounding alone. On the other hand, Fig. 12 shows that a 100-c/sec pure tone changed 10 percent, or five times as much as the complex tone, for this same loudness change. Therefore, the pitch of the complex tone at the higher loudness values is 8 percent higher than that of the pure 100-c/sec tone. When these tones are sounded alternately on this high loudness level, it is found that the pure tone actually is lower in pitch by approximately the amount indicated in Fig. 12. However, if the two are sounded together they are not discordant, for under such circumstances a new tone different from either is formed. In this new tone the fundamental is strengthened and the pitch of the resulting combination is slightly lowered so that its pitch is about 3 percent below a pitch of 100 c/sec.

So one cannot conclude that, if two tones sounding separately are slightly different in pitch, when they are sounding together they will be discordant. They may or may not be so. Conversely, two tones sounded alternately may have the same pitch but may be discordant when sounded together. A combination will be harmonious or not, depending upon the frequencies rather than on the pitch of the components. These facts concerning the relation between pitch and frequency will, when thoroughly understood, no doubt have a bearing upon the composition and rendition of music and also upon the tuning of musical instruments.

Quality

The quality of a musical tone is a characteristic that is less easy to define than are loudness and pitch. One might incorporate the latter two characteristics in such a definition and say that quality is the characteristic which enables one to judge that two tones are dissimilar while still having the same loudness and pitch. Quality is frequently defined as that characteristic of the sensation which enables the listener to recognize the kind of musical instrument producing the tone—that is, whether it is a cornet, a flute or a violin. But it is commonly known that the quality of tones coming from two different violins may be greatly different, and we have no adequate language to express this difference. Such language is greatly needed. Very little experimental work of scientific value has been done on quality. It is true that numerous attempts have been made to find the overtone structure of musical tones, but such work is very different from that of obtaining judgment tests of various types of musical tones under scientific control.

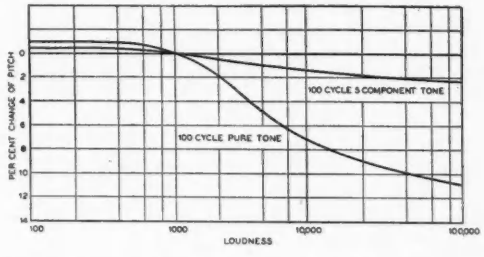


FIG. 12. Pitch *vs.* loudness for 100-c/sec five-component tone and 100-c/sec pure tone.

By analogy with what has been said regarding pitch and loudness, it seems advisable to choose scales and a reference tone for making comparisons of quality. It is well known that changes in overtone structure produce corresponding change in the quality, but it is not so well known that changes in loudness or pitch, without in any way changing the overtone structures, will also produce changes in quality. For example, when a violin sound is sent over a high quality reproducing system in such a way that its overtone structure is unchanged but its intensity amplified so as to have a loudness level 10 or 20 db higher than the sound coming directly from the violin, then its quality is greatly changed. Consequently, in selecting a reference tone for quality comparison, we must use apparatus such that the intensity, the frequency and the overtone structure of the reference tone can be varied throughout the audible range.

A tone generator that is sufficiently flexible to do all these things is difficult to build, but one is presented (Fig. 2) which we call a *tone synthesizer*. We have planned for a long time to have such an instrument for studying the quality of tones but succeeded in the actual construction of it only about five years ago, just before the war started. It does not completely fulfill the aforementioned conditions, since it is limited to overtone structures that have harmonic overtones. This is the type of overtone structure most frequently met in all types of musical instruments. It was designed to cover the audible ranges of frequencies and intensities. One hundred components are available. As it is arranged today, the fundamental is 50 c/sec and the highest component is 5000 c/sec. By changing the speed of the rotating shaft and some of the filters the fundamental may be raised to 100 or 150 c/sec, and then the highest component will be 10,000 or 15,000 c/sec. Each component tone can be called pure to the extent that any overtone in it is 55 db, or more, below the fundamental. That is, the overtone intensities are about one-millionth that of the fundamental tone desired. This severe requirement on tone purity was desired so that the synthetic overtone structure indicated by the positions of the white knobs (Fig. 2) would be correct within the range of the attenuators, namely 50 db.

As we slide a white knob from the top to the bottom, the corresponding tone is attenuated 50

TABLE III. Demonstration 10.

Clarinet: fundamental, 200 c/sec

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	∞	4.5	∞	0	25	6	11	13	13	16	25	23	17	16	23	29	19	23	28	33	33	33	35	37

Cello: fundamental, 100 c/sec

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
7	0	0	0	2	8	11	13	17	21	26	32	43	47	31	28	26	28	31	33	36	39	43	46	42	42	41	41	41	40	38	38	39	38	40	42

37	38	39	40	41	42	43	44	45
42	42	42	43	42	43	44	45	45

Flute: fundamental, 600 c/sec

1	2	3	4
0	3	15	38

Organ, diapason pipe: fundamental, 50 c/sec

1	2	3	4
0	10	20	30

Organ, diapason pipe: fundamental, 400 c/sec

1	2	3	4
0	10	20	30

Piano: fundamental, 100 c/sec

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
10	0	5	0	8	10	13	5	11	13	0	24	15	20	22	22	22	23	27	21	17	17	21	23	13	18	24	29	30	40	40	40	42	41	45	46

37	38	39	40	41	42	43	44	45	46	47	48
44	43	48	46	42	43	41	47	46	44	48	50

Bell: fundamental of series, 50 c/sec; pitch, 300 c/sec

3	7	11	17	19	24	29
35	7	3	0	5	13	27

Bass drum: fundamental, 50 c/sec

1	4	7	9	11
0	20	20	30	30

db. So for the 100 components there are 100 attenuators and 100 knobs. The position of these knobs gives the overtone structure on the kind of scale mentioned above. The tones are generated by the rotation of a single shaft on which the tones are magnetically recorded.

We will now construct a few synthetic tones on the synthesizer. First a tone with all the components of uniform intensity will be produced. We probably ought to call this a physicist's tone. We will use first the piano timing and then the organ timing, as shown in Fig. 3.

Demonstration 8.—With all the 100 components equally intense, the two tones—with piano timing, then with organ timing—were sounded.

The next two tones are produced with the white knobs on a sloping line.

Demonstration 9.—The two tones—with organ timing, then with piano timing—were sounded.

I suggest that these two tones are worthy of inclusion in musical productions.

Demonstration 10.—Tones of the musical instruments listed in Table III were imitated on the synthesizer. The table shows the overtone structure used in each case. The secret of imitating the impact instruments is in getting the proper starting and stopping time pattern.

With this tone synthesizer it is possible to produce almost an infinite variety of musical tones. We are just beginning a study to find the combinations that we think will have interesting possibilities. As such studies progress they will in my opinion have considerable influence upon our future music.

Thus we see that of the three characteristics of the sensation experienced by one listening to a musical tone, the loudness depends principally upon the intensity, but may change markedly with changes in the frequency or in the overtone structure. The pitch depends principally upon the frequency; but changes in the intensity produce small changes in the pitch, and certain types of changes in the overtone structure may produce large changes in pitch. The quality depends principally upon the overtone structure and the growth-decay time characteristic, but is also changed by large changes in the intensity and in the frequency. Changes in the wave form produced by changes in the phases between components produce no noticeable effects either in the loudness, pitch or quality of the tone.

Functions of University Science Departments

WHAT, then, in short should be our basic strategy with regard to the universities and research? First, we must look to them for the supply of the research workers required in all branches of science and for industry, for government institutions and departments, and for fundamental research at the universities themselves. Second, we must look to them ultimately for the supply of leaders in all walks of society competent to apply scientific knowledge to the service of industry or of the nation as a whole. Third, we must look to them to play a vital part in that work of adult education through which alone we can hope for a society in which policies and plans based on scientifically ascertained facts can be assured of reasoned and general support. Finally, these teaching functions must be in balance with the equally vital function of research extending the bounds of knowledge.

. . . teaching must be in vital touch with research, and . . . we may have to consider more carefully to what extent the two functions can be combined in the same staff. That there must be the vital contact is not denied, nor the value to the research worker himself of attempting to expound to others the significance of the field in which he is working. What we are concerned with is raising the standard of teaching and with giving to the really great teacher with a genuine talent for exposition and for inspiring others the full scope and encouragement that he deserves. Such teachers are not necessarily great investigators also, and one reason for the inadequate appreciation of science is certainly the failure of scientific workers themselves to accord fitting status and prestige to the great expositor in their ranks. We would do well, as Sir. J. J Thomson urged, to pay far greater regard in our appointments to teaching posts, . . . to the powers of the candidates to present a subject in a clear and attractive way.—Editorial, *Nature* **154**, 286 (1944).

The Early Years of Radioactivity

G. E. M. Jauncey
Washington University, St. Louis 5, Missouri

1. The Discovery of Radioactivity

THE discovery of x-rays was sudden. These rays were unknown to all physicists in the early morning of November 8, 1895; but, by late evening, Roentgen knew that he was dealing with a new kind of rays which he called x-rays. The story of the discovery of x-rays has recently been told again by E. C. Watson,[1] Otto Glasser,[2] and myself.[3]

In contrast, the recognition of the true nature of the phenomena which, we now know, constitute radioactivity came as the result of the accumulation of many seemingly disconnected facts over a period of about seven years. The accumulation began with Henri Becquerel's paper in the *Comptes rendus* for February 24, 1896. The title of this paper was "On the radiations emitted by phosphorescence." This paper was followed by five other papers, the last one of which appeared on May 18, 1896 under the title "Emission of new radiations by metallic uranium." The titles and the text of Becquerel's six papers on uranium and its compounds during this period show the gradual emergence of the idea that the penetrating and invisible radiations emitted by these substances are not connected with phosphorescence but constitute a hitherto unknown type of radiation emitted by the element uranium itself.

In 1898, Marie Sklodowska Curie[4] concluded that uranium radiation was an atomic phenomenon. Also, in 1898, Pierre and Mme. Curie introduced the term *radioactivity*. In 1902–3 Rutherford and Soddy published their epochal series of papers on the theory of the disintegration of radioactive atoms. This theory immediately brought about a grand synthesis of the facts of radioactivity. In conning the journals, I have been amazed at the inspired perspicacity and the intrepid perseverance with which the physicists of the period picked their way through the baffling maze of the phenomena of radioactivity.

This year of 1946 is the fiftieth anniversary of Becquerel's finding of the first indications of radioactivity. The present article is in commemoration of this event. I have set myself the same four tasks, *mutatis mutandis*, as in my previous paper.[3] However, I shall lay more stress on how radioactivity advanced during the period under review—1896 through 1904—and on the mental concepts of the physicists involved. I regret that lack of space has forced me to pass over the names of so many workers who made valuable contributions to the advance of physics in this period. The names of some 65 workers in addition to the famous five are mentioned in Rutherford's *Radioactivity* (1904).

For ease in reading, references are given in the text of the following sections. In this kind of article, dates are more important than volume and page numbers.

2. Becquerel Rays

Becquerel's account of his gradual unfolding of the fact that uranium and its compounds emit invisible and persistent radiations which pass through substances that are opaque to ordinary light is given in six papers appearing in the *Comptes rendus* during the period February 24, 1896 through May 18, 1896. After a description of the first paper I shall treat the remaining five papers together.

Phosphorescence

Early in 1896 many physicists, including J. J. Thomson and Henri Poincaré, believed that the x-rays emitted by the anticathodic wall of a Crookes tube were connected with the phosphorescence (we call it fluorescence in 1946) of this wall. Also it was known that x-rays caused

[1] E. C. Watson, *Am. J. Physics* **13**, 281 (1945).
[2] O. Glasser, *W. C. Roentgen* (Thomas, Springfield, Ill., 1945).
[3] G. E. M. Jauncey, *Am. J. Physics*, **13**, 362 (1945).
[4] In 1898, Mme. Curie wrote her articles in the *Comptes rendus* under the name of Sklodowska (or S.) Curie.

phosphorescence in certain crystalline materials such as barium platinocyanide. It therefore seemed worth while to search for penetrating radiations from substances that were phosphorescent under the action of x-ray or light. On February 10, 1896, Charles Henry read a paper before the French Academy of Sciences on "The increase of the photographic effect of x-rays produced by phosphorescent zinc sulfide." A week later, Niewenglowski reported that commercial calcium sulfide when exposed to sunlight emitted some rays which were able to penetrate black paper and affect a photographic plate. However, these results have not stood the test of more rigorous experiment.

On February 24, 1896, Henri Becquerel reported to the French Academy that a thin layer of the double sulfate of uranium and potassium after exposure to direct sunlight for several hours emitted rays which penetrated black paper and affected a photographic plate. In the experiment, half of one side of the black-paper envelope containing the plate was covered with the layer of the double sulfate. The developed negative was much *blacker* where it had been under the layer than elsewhere. When a coin or a sheet of metal perforated with a design was interposed between the layer and the black paper, the developed negative showed less blackening under the solid parts of the objects interposed. Similar results were obtained when a thin sheet of glass was placed between the layer and the envelope. The glass precluded any chemical action due to vapors emitted by the double sulfate.

Persistent Radiations

So far there was nothing in particular to distinguish the effect found by Becquerel and the effects found by C. Henry and Niewenglowski. However, Becquerel soon found that diffuse sunlight stimulated the phosphorescence of the double sulfate of uranium and potassium about as well as direct sunlight. He next fastened a thin button of the double sulfate to the outside of the aluminum shutter or slide of a plate holder containing a photographic plate and exposed the button and plate holder to sunlight. The developed negative showed black under the button. Another trial in which the button was absent gave no blackening of the negative, thus showing that the holder was light-tight. The blackness under the button was reduced when a thicker aluminum shutter was used. A number of plate holders with their attached buttons having been prepared for further experiments, it so happened that the sky was overcast and the sun shone intermittently for several days. During these days Becquerel kept the plate holders in the darkness of a drawer. Perhaps because of impatience after waiting four days for the sun to shine, Becquerel developed one of the photographic plates. To his astonishment the part of the negative under the button was unusually black instead of being faintly black as he had expected. Other negatives showed similar results. He then assembled a photographic plate, a plate holder with an aluminum shutter, and a button of double sulfate completely in the dark. After five hours the developed negative showed blackness under the button. This startling result was obtained on *March 1, 1896.*

Becquerel ventured the opinion that the invisible and penetrating rays he had found were analogous to the radiations recently found by Lenard and Roentgen. It must be remembered that in 1896 the word "ray" still meant a narrow beam of radiation or of ethereal vibrations. It was not realized that a "ray" might be either a narrow bundle of waves or a moving particle. No concept of a particle smaller than the hydrogen atom was in anyone's mind at the time. The electron had not yet been discovered.

Becquerel continued his probing into the mystery of uranium salts even though he believed that the rays he was studying were caused by phosphorescence. It is no wonder that his mind was fixed on this idea since both his father, Edmond Becquerel (1820–1891), and his grandfather, A. C. Becquerel (1788–1878), had worked on the problem of phosphorescence produced by ultraviolet and visible light. In particular, Edmond Becquerel had studied the visible phosphorescence of the salts of uranium. The experimenter must have a working hypothesis in order that he may be able to pick his way through the maze of phenomena with which he is confronted. I have quoted Faraday on physical theory in my previous paper.[3] The experimenter must heed Faraday's warning never to venerate a

hypothesis to such an extent that it becomes a prejudice and so interferes with clear-sighted judgment.

Uranic nitrate ceases to be visibly phosphorescent when it is in solution or is dissolved in its water of crystallization. Becquerel accordingly heated a crystal of the nitrate in the dark in a small closed glass vessel until it dissolved. The liquid was then allowed to cool and to recrystallize. Yet this recrystallized nitrate, though at no time exposed to light, emitted radiations that affected a photographic plate as strongly as when the crystal had been exposed to light. Becquerel also found that uranous salts, which are not visibly phosphorescent in ordinary light, emit invisible rays as do the uranic salts. Then he found that solutions of uranium salts, although not visibly phosphorescent, emit invisible rays. Finally he found that uranium metal emits invisible rays. He stated that this was the first example of a metal presenting a phenomenon of the order of an invisible phosphorescence. He further stated that the intensity of the invisible radiation emitted by a uranium compound was proportional to the fraction by weight of the uranium in the compound.

But the persistence of the invisible radiation of uranium and its compounds puzzled Becquerel. First he found that the invisible radiation persisted undiminished for a few days although the visible phosphorescence persisted for less than 0.01 sec after the exciting light had been shut off. He constructed a light-tight apparatus containing a button of the double sulfate of uranium and potassium. He could slip in a photographic plate under an aluminum sheet whenever he pleased. By such a device he found that the invisible rays persisted undiminished for six days, then for 15 days, for two months, eight months, and finally for three years. This last figure was reported in the *Comptes rendus* for March 27, 1899. Meanwhile, *Nature* for May 6, 1897 reported that Elster and Geitel had found that uranium salts may be kept in the dark for months without the radiations ceasing, and that the radiations are not noticeably promoted by sunlight. Elster and Geitel also stated that the rays could not be attributed to hyperphosphorescence of exceedingly long duration.

Reflection, Refraction and Polarization

The experimenter is often bedeviled by wrong turnings and blind alleys in the maze of phenomena that he is studying. Becquerel was thus bedeviled when he seemingly found evidence for the reflection and refraction of Becquerel rays. Roentgen had just announced that x-rays were neither reflected nor refracted. All other ethereal waves known at the time could be reflected and refracted. The misbehavior of x-rays in this regard greatly puzzled the physicists then searching for the nature of x-rays. Roentgen suggested that perhaps x-rays were longitudinal vibrations in the ether. It was a comfort to learn from Becquerel that his rays were well behaved and capable of reflection and refraction.

In one experiment Becquerel constructed a container out of a glass tube sealed at one end to a cover glass. The substance to be tested was placed in the tube, and then a second cover glass was cemented to the other end of the tube with paraffin. The tube stood upon an aluminum plate, 2 mm thick, resting on a photographic plate in a black-paper envelope. With a uranium salt in the tube a black patch bounded by a blacker ring was obtained on the plate. The ring appeared under the glass wall of the tube. The negative showed a white zone outside the ring. Becquerel took this to indicate total reflection of the invisible rays at the glass-air surface. He likened the result to internal reflection of light in a jet of water in the familiar laboratory experiment and stated that, if the phenomena of reflection and refraction had not been shown by other experiments, they would have been shown by this test alone.

Watson's article[1] includes a translation of Roentgen's first paper on x-rays. According to Sec. 7 of that paper, Roentgen found that a substance is just as transparent to x-rays when in the form of a solid plate as when in powdered form, provided that the mass per unit area is the same in both cases. Roentgen took this as sufficient evidence to show that x-rays were neither reflected nor refracted. Becquerel performed a similar experiment with his rays and obtained a result similar to that for x-rays. Becquerel, however, was unwilling to concede that his rays could be neither reflected nor

refracted. He gave greater weight to the experiment described in the preceding paragraph. In this he was following his predilections, and in 1896 other physicists, including J. J. Thomson, followed Becquerel.

Becquerel also claimed in 1896 that he found evidence of greater absorption of his rays in crossed than in parallel tourmalines. However, on repetition of his experiments on reflection, refraction and polarization three years later, in 1899, he was unable to confirm his previous results. Others likewise were unable to confirm Becquerel's results of 1896, so that from 1899 on it was believed that Becquerel rays were not capable of reflection, refraction or polarization. But much had happened between 1896 and 1899. X-rays were better understood in 1899 and the electron had been discovered in 1897.

Discharge of Electrified Bodies

In the first half of 1896 Becquerel reported that his rays had the property of discharging an electroscope. In this the Becquerel rays were similar to x-rays and to cathode rays-in-air (Lenard rays). The discharge method permitted more quantitative measurements of the intensity of Becquerel rays to be made than did the photographic method. However, much had to be learned about the discharge of electrified bodies in air and other gases before the electrical method of measuring Becquerel rays could be considered as reliable. In the early experiments the importance of the saturation current and of proper shielding was not realized. The work of J. J. Thomson and others on the ionizing effect of x-rays on gases gave a much better understanding of how to use the electrical method properly. In November 1896 Becquerel found that air which had been passed over uranium in the dark was electrically conductive.

Absorption

In March 1896 Becquerel, using the electrical method, measured the absorption of his rays in various substances. He placed a shallow tray containing a uranium compound below the leaves of an electroscope, and over the tray he placed a sheet of the substance to be tested. He found that a sheet of aluminum, 0.10 mm thick, reduced the rate of discharge to 0.26 of the rate without the aluminum. For a 0.09-mm sheet of copper and a 0.035-mm sheet of platinum, the fractions were 0.32 and 0.27, respectively. When x-rays were used instead of Becquerel rays, the sheets of aluminum allowed intense x-rays to pass whereas the sheet of copper was almost opaque to x-rays. The absorption law for Becquerel rays was thus very different from that for x-rays.

If the Becquerel rays are homogeneous, the fraction for the aluminum and copper sheets superposed should be 0.26×0.32, or 0.083. The actual fraction found was 0.21. Becquerel concluded that his rays were not homogeneous. A similar conclusion had been drawn by J. J. Thomson for x-rays.

3. The Advance in 1898

The period from the end of May 1896 till the end of 1897 was one of stagnation in the field of Becquerel rays. Even Becquerel himself left the field and published papers on the Zeeman and Faraday effects. During this period the startling fact of the persistence of the invisible rays from uranium and its compounds slowly emerged. The Curies came into this field in early 1898.

Thorium

In the first half of 1896, Becquerel, using the photographic method, had tested phosphorescent specimens of calcium, zinc and strontium sulfides for persistent invisible radiations and found no such effects. In May 1896 Becquerel attributed the persistent rays from uranium and its salts to the uranium present. Yet he did not go a step further and admit that the emission of the rays might be an atomic property of uranium. He made no systematic search for other radioactive[5] elements. The idea of making such a search occurred simultaneously to G. C. Schmidt in Germany and Mme. Marie Sklodowska Curie in France. Schmidt, in the *Annalen* for April 15, 1898, and Mme. Curie, in the *Comptes rendus* for April 12, 1898, announced that thorium compounds were racioactive. Of the compounds of many elements tested, only those of uranium

[5] The word "radioactive" came into use in 1898.

and thorium possessed the properties of blackening a photographic plate after passing through a sheet of aluminum and of imparting electrical conductivity to gases near or passing over them. Schmidt took the latter property as the crucial test for radioactivity. Several phosphorescent or fluorescent substances under the proper stimulus emitted radiations that could penetrate aluminum and blacken a photographic plate, but only the radiations from compounds of uranium and thorium imparted conductivity to air. The absorption of thorium rays in sheets of metal and other solids was similar to that of uranium rays. Schmidt, using Becquerel's method of enclosing a thorium compound in a glass tube closed at each end with a cover glass, found photographic evidence for the refraction of thorium rays. However, he stated that the rays were diffusely reflected (scattered). He found no evidence for polarization by parallel and crossed tourmalines. The discharging action of thorium rays was much stronger than that of uranium rays.

Mme. Curie tested the compounds of both uranium and thorium by placing a uniform layer of the substance to be tested on the lower of two parallel metal plates 3 cm apart. A difference of potential of 100 v was maintained between the plates, and the current was measured with an electrometer. Two minerals of uranium, pitchblende and chalcolite, were found to be much more active than uranium itself.

The importance of the discovery that thorium as well as uranium is radioactive is not properly emphasized in books on radioactivity. Before 1898, radioactivity was considered as a unique and anomalous property of uranium. The radioactivity of thorium removed uranium from its unique position in this regard. Uranium was also unique in being the element of highest atomic weight then known. It is astonishing that in 1896 and 1897 it did not occur to someone to test a compound of thorium, the element of next highest atomic weight then known, for radioactivity.

Polonium

As a result of her experiments on compounds of uranium and thorium, Mme. Curie concluded that the radioactivity of these compounds is an *atomic property* of the uranium and thorium present. Becquerel had stated in 1896 that the invisible radiation from uranium was a property of the element. Mme. Curie's conclusion in 1898 that the invisible radiation was an atomic property was a distinct advance in the thinking of those days. This idea led the Curies to the view that pitchblende and chalcolite, whose radioactivities are greater than those calculated from the proportion of uranium in each mineral, must contain a new and unknown element more radioactive than uranium. In order to test this conclusion they made artificial chalcolite out of all the elements which at that time were known to be present in the mineral. The artificial mineral showed the radioactivity to be expected from the uranium present. Mme. Curie and her husband, Pierre Curie, then set out to search for this new element. In addition to the conclusion just mentioned, the Curies assumed that the electrical test for radioactivity was far more sensitive in searching for minute quantities of a new radioactive element than the methods of chemical and spectral analysis. These were very bold assumptions at the time.

The device of parallel metal plates described in the preceding subsection was used. A thin layer of the substance to be tested was spread over a certain area of the lower plate and the current was measured. The substance was removed, a standard layer of a uranium compound was substituted and the current measured. The ratio of the currents gave the activity.

Pitchblende is a mineral containing barium, bismuth, and various other elements besides uranium. The Curies proceeded to the decomposition of pitchblende and its separation, by chemical analysis, into its constituent elements, testing each preparation for its activity. They found that the bismuth sulfide and the barium sulfide separated from pitchblende were both strongly radioactive. These active preparations were soon known as "radiferous bismuth" and "radiferous barium." The Curies found that, if the radiferous bismuth sulfide is heated in a vacuum at 700°C, the active part, being the more volatile, tends to deposit on the cooler parts of the tube. The sublimate thus obtained had 400 times the activity of uranium. Since no chemical substance out of a large number examined behaved in a similar manner, the Curies

ventured the opinion that they had partially isolated a new element, for which they suggested the name "polonium" in honor of the country of Mme. Curie's birth. The results and tentative conclusions were reported to the French Academy on July 18, 1898. However, at that time, the existence of a new element in small quantities was accepted only if its spark spectrum could be obtained. Since the Curies presented no such evidence, the radioactive evidence was received with reservations. This was natural since radioactivity was a new and ill-understood branch of physics. The new "element" resembled zinc in its chemical properties but differed from it in being radioactive.

Radium

The Curies and Bemont then examined the radiferous barium which had been separated from pitchblende. Ordinary barium is not radioactive, and so the radiferous variety must, on the Curies' hypothesis, contain a new radioactive element. Indications were that this new element was still more radioactive than polonium and that it chemically resembled barium. The new element was provisionally named "radium" because it possessed the property of radioactivity to an astonishing degree. The chlorides of barium and radium are soluble in water. Fractional precipitation of the chlorides by alcohol showed an increasing amount of radium in the precipitate, the final fraction possessing an activity 900 times that of uranium. This result was reported to the French Academy on December 26, 1898. At the same meeting Demarçay reported that at the request of the Curies and Bemont he had subjected a sample of the radium chloride to spectroscopic examination and had found a strong new line (3814.8A) in the ultraviolet. Physicists and chemists accepted this evidence for the existence of a new element. The sample tested by Demarçay was of low activity and naturally showed the barium lines as well as the new line ascribed to radium.

The radiations from radium blacken a photographic plate and impart conductivity to air. If one side of a very thin sheet of aluminum is covered with a thin coating of a polonium or a radium compound and the other side of the sheet is covered with a thin layer of barium platinocyanide, the cyanide fluoresces faintly. The Curies and Bemont pointed out that this device acts as a source of light to which apparently no energy is supplied. In 1899 Crookes suggested that the radioactive substances can abstract energy from the surrounding air or gas. If the more swiftly moving molecules impinging on the substance are released later at a much lower velocity, the energy released may be derived from the atmosphere.

Mme. Curie set herself the task of measuring the atomic weight of the new element radium. The successive fractionations of radiferous barium chloride contained an increasing proportion of radium chloride. At first all that could be done was to determine the average atomic weight of the barium and radium in the radiferous barium chloride. The method employed was to precipitate the chlorine in a known weight of the chloride with silver nitrate and to weigh the silver chloride obtained. First attempts gave an atomic weight of 146 as against 138 for non-radiferous barium. Mme. Curie reported on August 6, 1900 that as the result of prolonged fractionations a product had been obtained with an atomic weight of 174. Continuing this most tedious process of fractional crystallization, Mme. Curie was able to report on July 24, 1902 that she had obtained about 0.1 gm of radium chloride which on spectroscopic examination gave no evidence of barium and that the atomic weight of radium is 225 on the assumption that radium is a divalent metal. The anhydrous chloride is spontaneously luminous. Beginning with something like two tons of pitchblende, Mme. Curie had spent three and a half years in determining the atomic weight of radium. It is no wonder that J. J. Thomson, in *Harper's Magazine* for January, 1903 referred to the "indefatigable zeal and experimental skill" of the Curies.

4. The Radiations

Before 1899 physicists were concerned with the search for new radioactive substances. But, in the *Philosophical Magazine* for January 1899, Rutherford turned his attention to a more careful study of the radiations themselves. He used an

electrical measuring device similar to that used by the Curies. A powdered uranium salt or powdered uranium metal was placed on the lower plate of a condenser, and foils of various metals were placed over the uranium. Using aluminum foil, 0.0005 cm thick, Rutherford found that the curve obtained by plotting the current as a function of the number of layers of the foil could be expressed as the sum of two exponential curves. For the first three layers one exponential curve was followed, while after about six layers another exponential curve was followed. In the equation $I=I_0e^{-\mu d}$, $\mu=1600$ cm^{-1} for the first three layers and $\mu=15$ cm^{-1} for the layers between about the sixth layer and the next 100 layers.

This experiment showed a remarkable difference from experiments with x-rays. J. J. Thomson had found that μ decreased continuously as more layers of aluminum were added. This was interpreted to mean that x-rays were heterogeneous and that, on the view that they were undulatory motions in the ether, the x-rays emitted by an x-ray tube consisted of a continuous spectrum of wave-lengths.

At the beginning of 1899, Rutherford's results were taken by himself and by others to mean that the Becquerel rays from uranium consisted of two kinds of radiation differing chiefly in their penetrating powers. The electron had been isolated by J. J. Thomson in the latter part of 1897. But, although cathode rays were accepted as streams of charged particles, the word "rays" in the term Becquerel rays still implied ethereal radiation. An exponential law of absorption of such radiation was taken as proof of monochromatic radiation. Rutherford named the radiations with $\mu=1600$ cm^{-1} and $\mu=15$ cm^{-1} "α"- and "β"-radiations, respectively. He measured the "opacity" μ for β-radiation in various metals and found that it followed the order of atomic weights. His results were not in agreement with Becquerel's finding that copper was less opaque than aluminum for uranium radiation.

The β-radiation was found to be about as penetrating as the x-rays from an average tube, but much less than the x-rays from a hard tube. Since the α-radiation was so much more easily absorbed, Rutherford offered the suggestion that it might be a secondary radiation set up at the surface of a uranium salt by passage of the β-radiation in exactly the same way as diffuse radiation (secondary x-rays and secondary cathode rays, as we know in 1946) is produced at the surface of a metal by x-rays. However, Rutherford stated that there was not sufficient evidence at that time to decide the question. In 1902 Rutherford and Soddy found that the α-ray activity of pure uranium could be separated from the β-ray activity of uranium X. This showed that the α-rays were not secondary β-rays.

In view of what happened some five years later, Rutherford's experiments on the absorption of α-radiation in gases are of interest. The ionization chamber consisted of two horizontal plates. A circular opening, 7 cm in diameter, was cut in the lower plate and covered with aluminum foil, 0.0005 cm thick. The lower plate was maintained at a potential of 60 v, and the upper plate was connected to an electrometer and a grounding key. The distance between the two plates was 1.5 cm. The distance r of the lower plate above a third plate could be changed by means of a screw. A layer of uranium salt was placed on the third plate. If the absorption in the gas is exponential, if the number of ions produced in the gas is proportional to the absorption in the gas and if the radiation is assumed to travel vertically upward, the ionization current is given by

$$I=\text{const.}\times e^{-\mu r}. \qquad (1)$$

The results are shown in Fig. 1. Since the experi-

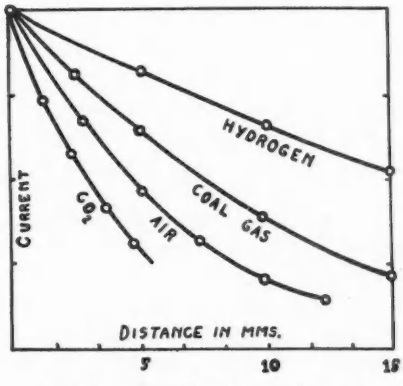


FIG. 1. Absorption of α-rays in gases [*Phil. Mag.* **47**, 126 (1896)].

mental values fall so well on an exponential curve, it was assumed for the next five years that the fundamental law of absorption for α-radiation was exponential.

Rutherford found that more β-radiation was emitted by a thick layer of uranium than by a thin layer. The amount of α-radiation, however, remained the same. Rutherford further found that the ionization current depends upon the surface area for a given uranium salt of given weight. A small crystal of uranium nitrate was dissolved in water and the water then evaporated so as to deposit a thin layer of the salt over the bottom of the dish. This gave quite a large current which was caused by the α-radiation. The ionization current therefore depends on the state of division of the compound. Rutherford suggested that the powerful radiation obtained from pitchblende by the Curies might be partly caused by the very fine state of division of the substance rather than by the presence of a new element, polonium.

A few months after Rutherford's announcement that uranium rays consisted of α- and β-radiations the corpuscular nature of the β-radiation was revealed by experiments made almost simultaneously in Germany and France. Because Elster and Geitel had noted that a magnetic field altered the conductivity imparted to air by radium rays, Geisel was led to investigate the effect of a magnetic field on the radiations. Giesel's results were reported in the *Annalen* for December 1899. The radioactive preparation was placed in a small vessel between the poles of an electromagnet, and the vessel was arranged so as to give a pencil of rays that was approximately perpendicular to the field. The rays produced a small luminous patch on a fluorescent screen. When the electromagnet was excited, the fluorescent zone was observed to broaden out on one side. When the field was reversed, the extension of the zone was in the opposite direction. The deviation of the rays was in the same direction and of the same order of magnitude as that for cathode rays. Becquerel a little later confirmed Giesel's results by the photographic method.

In the *Comptes rendus* for January 8, 1900, P. Curie reported that the rays from radium consisted of two kinds, one apparently non-deviable and easily absorbed (now known as α-rays) and the other penetrating and deviable by a magnetic field (now known as β-rays). P. Curie found that, of the deviable rays, the most deviable were the least penetrating. On the same date, Mme. Curie reported that there is a fundamental difference in the absorption law for the deviable rays and that for the nondeviable rays. For the deviable rays the intensity I in the equation $I=I_0e^{-\mu d}$ falls *more slowly* with increasing thickness d than the exponential law requires, whereas for the nondeviable rays the intensity I falls *more quickly* than the exponential law requires. Mme. Curie stated that this singular law for the nondeviable rays differs from that for any other known radiation.

In the early part of 1900, the exponential law was accepted as the fundamental law of absorption. The β-rays were known to be particles. Even so, any departure from the fundamental law was ascribed to lack of homogeneity. Since, in the thinking of the time, the less penetrating rays were weeded out by the absorption, it was exceedingly difficult to understand how the parameter μ could *increase* with d, as Mme. Curie had found that it did for the nondeviable rays. The α-rays were not yet known to be particles.

In February 1903 Rutherford reported that he had measured both the magnetic and electrostatic deviation of α-rays. Before 1903 the α-rays were considered nondeviable by a magnetic field, but this was owing to the very small deviation produced even by very strong magnetic fields. In Rutherford's experiment, the α-rays from a thin layer of radium of activity 19000 passed upwards through a set of parallel plates of height 3.70 cm, width 0.70 cm, with an average air space between the plates of 0.042 cm, into a testing vessel. The distance of the radium below the plates was 1.4 cm. A current of dry hydrogen was passed between the plates and over the radium to carry off the emanation. With a field of 8370 gauss there was a large reduction in the rate of discharge of the electroscope. A similar system of plates was used for the electrostatic deflection but with alternate plates connected to the positive terminal of a 600-v battery and the other plates connected to ground. Care was taken to avoid sparking. The value of e/m ob-

tained was 6×10^8 emu gm^{-1} with a velocity of 2.5×10^9 cm sec^{-1}. The deviations were in such a direction as to indicate a positive charge on the α-particle. The interpretation of a value of e/m equal to about half that for the hydrogen ion in electrolysis was uncertain at the end of 1904.

In 1903, Crookes, and independently Elster and Geitel, found that a fluorescent screen of zinc sulfide subjected to α-rays showed scintillations when examined with a microscope. Rutherford suggested that the method of scintillations might be used for counting α-rays.

In the *Comptes rendus* for April 30, 1900, Villard reported finding very penetrating rays that were nondeviable by a magnetic field. The α- and β-rays were completely absorbed by placing lead of thickness 1 cm over the radium bromide used. Villard used the photographic method. These nondeviable but very penetrating rays came to be known as γ-rays. They are best investigated by the electrical method. The γ-rays from a radium preparation produce fluorescence in a barium platinocyanide screen. Villard's results were quickly confirmed by Becquerel. In July 1902 Rutherford reported that thorium salts as well as radium salts emitted γ-rays. In 1902 Rutherford suggested that γ-rays might be electrons moving with a speed approaching that of light. However, in his *Radioactivity* (1904) he suggested that, since x-rays are ether pulses produced by the sudden stoppage of electrons, γ-rays might be ether pulses produced by the sudden starting of β-rays. In 1904 it was believed that γ-rays always occurred along with β-rays. On the whole, however, not much attention had been paid to γ-rays by the end of 1904.

5. Emanations

In January 1899 Rutherford reported that the electrometer readings obtained when a layer of a thorium salt was placed on the lower plate of the ionization chamber in the Curie apparatus were capricious. The readings would sometimes differ by a factor of five when thorium nitrate was used. Some months later Owens traced these capricious readings to the movement of air over the thorium compounds. He replaced the lower plate of the ionization chamber by a plate with a shallow circular depression in it. The thorium compound could be placed in this depression and the layer covered with sheets of various substances. Air was then blown through the ionization chamber. When the covering was a sheet of paper or of aluminum, the ionization current was less when the air was in motion than when it was at rest. If several sheets of aluminum foil were used, the reduction when air was blown through the apparatus was less than when only one sheet was used. When, however, the thorium compound was covered with mica, the actual ionization current was greatly reduced but remained practically constant whether the air was at rest or in motion.

Rutherford reported the results of further investigations in January 1900. He put forward the view that the thorium compounds, in addition to the α- and β-radiations, gave out a large number of radioactive particles from the mass of the active substance. This "emanation" could pass through considerable thicknesses of paper. The radioactive particles gradually diffused through the gas in its neighborhood and became centers of ionization. The fact that the effect of air currents was only slight with thin layers of thorium oxide was said to be caused by the preponderance of the ionization current due to the ordinary radiation over that due to the emanation. With a thick layer of thorium oxide, the ionization current due to the ordinary radiation was practically that due to a thin surface layer since the radiation could only penetrate a short distance through the salt. On the other hand, the emanation was able to diffuse from a distance of several millimeters below the surface of the compound, and the ionization current due to it became much greater than that due to the radiation alone.

Shortly after the discovery of the thorium emanation, Dorn found that radium compounds also give off radioactive emanations and that the amount given off is much increased by heating the compound. Both thorium and radium emanations ionize the gas with which they are mixed and affect a photographic plate. Both diffuse readily through porous substances but are unable to pass through a thin plate of mica; both behave like a temporarily radioactive gas, mixed in minute quantity with the air or other gas in which they are conveyed.

In 1901 Rutherford and Brooks determined the rate of diffusion of radium emanation into air. A long brass cylinder, 73 cm long and 6 cm in diameter, was divided into two equal parts by a movable slide. Each half had a central electrode connected to an electrometer. Radium emanation was introduced into one half with the slide closed. After several hours the slide was opened and the ionization current in each half of the brass cylinder determined from time to time. From these results the coefficient of diffusion of radium emanation into air was found to be about 0.07 $cm^2\ sec^{-1}$. For ether vapor the value as given in tables was 0.077 $cm^2\ sec^{-1}$. The molecular weight of the ether vapor is 74, and so Rutherford and Brooks concluded that the molecular weight of radium emanation was greater than 74.

The coefficient of diffusion of thorium emanation into air was determined by another method and found to be 0.09 $cm^2\ sec^{-1}$. In 1903 Wallstabe measured the coefficient of diffusion of radium emanation into water and found it to be 0.066 $cm^2\ day^{-1}$. The coefficient for carbon dioxide into water was given in tables as 1.36 $cm^2\ day^{-1}$. This result confirmed the view that radium emanation behaved as a gas of high molecular weight. In his *Radioactivity* (1904) Rutherford concluded that the molecular weights of the two emanations were about the same and that each emanation behaved as a gas of high molecular weight (perhaps over 100).

In 1903 Rutherford and Soddy passed air containing radium emanation through a **U** tube and then through an ionization chamber. With the **U** tube at ordinary temperatures, an ionization current was shown. When, however, the **U** tube was surrounded by liquid air in a Dewar flask, the ionization current quickly fell to zero. Rutherford and Soddy then substituted a copper tube wound in the form of a helix for the **U** tube and liquid ethylene for liquid air. They found the condensation and volatilization temperatures for the radium emanation to be the same and equal to about −150°C. For thorium emanation they found that condensation commences at about −120°C and that very little of the emanation escapes condensation at −155°C. Rutherford and Soddy ascribed the behavior of thorium emanation to the very small number of emanation molecules in the condensing helix. They estimated about 230 molecules/cm^3 for thorium emanation as against 1.15×10^6 molecules/cm^3 for the radium emanation.

In 1899 Debierne obtained actinium from pitchblende by chemical separation. Four years later, he found that actinium gives out an emanation similar to the emanations of thorium and radium.

6. Radioactive Transformations

Until the latter part of 1899, the known radioactive substances were uranium, thorium, polonium and radium. As far as experiments showed at that time, each of these substances emitted radiations at a constant rate. Then the Curies in the *Comptes rendus* for November 6, 1899 reported the finding of "excited radioactivity" produced by radium on neighboring bodies. A little later, in the *Philosophical Magazine* for January and February 1900, Rutherford reported the finding of thorium emanation and of "excited radioactivity" produced by thorium. The "excited radioactivity" later became known as the "active deposit." The emanation and the active deposit were radioactive elements differing from the previously known radioactive elements in that their activities decayed with time. This was a new and surprising phenomenon.

Rutherford showed that, in the case of thorium, the active deposit was produced by thorium emanation and not by thorium itself. He also found that, if the air carrying the thorium emanation was subjected to an electric field, most of the active deposit occurred on the negative electrode. Further, if thorium emanation was introduced into a vacuum, there was no tendency for the activity to deposit on the negative rather than on the positive electrode.

Rutherford found that the activity of a platinum wire which had been exposed to thorium emanation could be partly removed by treatment with certain acids. For instance, more than 80 percent of it was removed by dilute sulfuric or hydrochloric acid. The activity was not destroyed by this treatment but was transferred to the solution. When the solution was evaporated, the activity remained behind in the dish. The active matter was deposited on the surface of

the dish, for it could be almost completely removed by scouring the dish with emery paper.

The activity produced on a metal rod or plate after *long* exposure to thorium emanation decays exponentially with the time after removal from

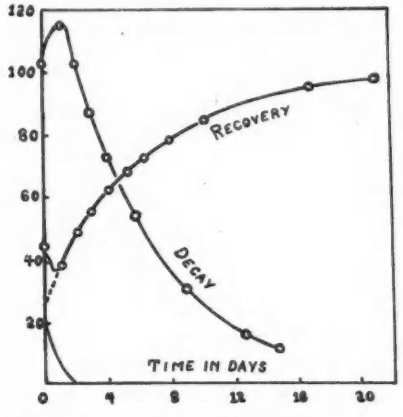


FIG. 2. Decay curve for Th X and recovery curve for thorium hydroxide [*Phil. Mag.* **4**, 380 (1902)].

the emanation, falling to half value in about 11 hr. This first radioactive decay curve was published in February 1900. It is the graph of I *versus* t in

$$I = I_0 e^{-\lambda t}, \tag{2}$$

where $\lambda = 0.063$ hr^{-1}, and I_0 is the activity measured by the electrical method immediately after removal of the rod from the emanation.

The amount of excited activity produced on a body at first increases with time, but reaches a maximum after an exposure of several days to thorium emanation. In this experiment, a metal rod was made the cathode in a closed vessel containing thorium oxide. The rod was removed at intervals for the short time necessary to test its activity and then replaced. In this case t is the time of exposure of the rod to the emanation and I_0 is the maximum activity measured. The graph obtained was represented by

$$I = I_0(1 - e^{-\lambda t}), \tag{3}$$

where $\lambda = 0.063$ hr^{-1}, the same value as in Eq. (2). Rutherford called this second curve a "recovery curve." These decay and recovery curves obtained by Rutherford in early 1900 constituted the first step in arriving at the theory of radioactive transformations, which was put forward in the latter part of 1902, nearly three years later.

In May 1900 Crookes announced the discovery of uranium X. This can be separated from uranium by treating a uranium solution with ammonium carbonate. On dissolving the precipitate of uranium carbonate in an excess of the ammonium carbonate a light precipitate was found to remain behind. This was filtered and constituted the uranium X. The active substance uranium X was probably present in very small quantity since no new lines were observed in its spectrum. Crookes stated that the uranium X contained all the photographic activity of the original uranium. This was taken at the time to mean that pure uranium was not radioactive. However, in July 1902 Soddy pointed out that although uranium is inactive photographically after the uranium X is removed, it is still active when tested by the electrical method. The photographic method measures the β-ray activity, and so Soddy concluded that pure uranium emits α-rays and uranium X emits β-rays.

In July 1900 Becquerel reported that barium could be made photographically very active by adding barium chloride to a uranium solution and precipitating the barium as sulfate. By a succession of precipitations, the uranium was rendered photographically almost inactive, while the barium was strongly active. The inactive uranium and the active barium were laid aside; but, on examining them a year later, it was found that the activity of the uranium had been completely regained while that of the barium had completely disappeared.

Rutherford and Soddy in September 1902 announced that thorium X could be obtained from thorium compounds by adding ammonia to a thorium solution. The thorium is precipitated, but a large amount of the activity as measured electrically is left behind in the filtrate, which is chemically free from thorium. This filtrate was evaporated to dryness and the ammonium salts driven off by ignition. A small residue was obtained which, weight for weight, was as much as several thousand times as active as the thorium from which it was obtained. This active constituent was named thorium X. The activity of the precipitated thorium was reduced to less than one-half its original value. On examining

the thorium a month later the thorium X was found to be no longer active while the thorium had completely regained its activity.

If the thorium hydroxide which is precipitated from a solution of thorium nitrate by ammonium hydroxide, as shown by

$$Th(NO_3)_4+4NH_4OH = TH(OH)_4+4NH_4NO_3, \quad (A)$$

is removed by filtering and the precipitate is dissolved in nitric acid, as shown by

$$Th(OH)_4+4HNO_3=Th(NO_3)_4+4H_2O, \quad (B)$$

and the process is repeated, then, after two such precipitations of $Th(OH)_4$, the thorium hydroxide is practically free from thorium. As quickly as possible after the removal of the thorium X, the thorium was tested for its radioactivity and the recovery curve was obtained, as shown in Fig. 2. At the same time the decay curve of the thorium X shown in Fig. 2 was also obtained. It is seen that both curves are irregular for the first two days. The activity of the thorium X increased at first, while the activity of the thorium diminished. Rutherford and Soddy decided to disregard these initial irregularities; they found that, on measuring time from $t=2$ days, the two curves were represented by Eqs. (2) and (3), where in this case the half-value was attained in about 4 days, or $\lambda=0.173$ day^{-1}. The step of disregarding the initial irregularities required inspired boldness. The two remarkable workers hazarded the hypothesis that the irregularities were caused by the presence of the active deposit. It had already been found that the active deposit came from the thorium emanation. Hence, so it was argued, the thorium emanation must come from thorium X and not from thorium itself.

We have seen that the active deposit from thorium decays to half-value in 11 hr. If the deposit is derived in two steps from thorium X and the thorium X is removed as it appears in the thorium over a period long compared with 11 hr, then, argued Rutherford and Soddy, it should be possible to obtain thorium not only free from thorium X but also free from the emanation and the active deposit. They therefore carried out 23 precipitations over a period of nine days. The thorium precipitate was then tested for its activity and Fig. 3 was obtained.

It will be noted that the new recovery curve for thorium, although showing no initial diminution in activity, does show an initial activity

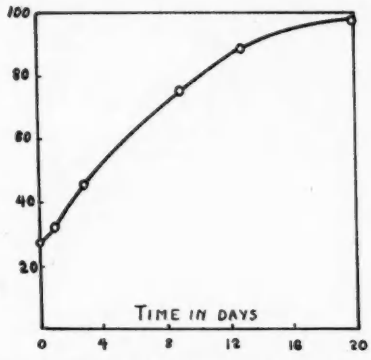


FIG. 3. Recovery of 23 times precipitated thorium hydroxide [*Phil. Mag.* **4**, 389 (1902)].

of about 25 percent of the final activity. Hence Rutherford and Soddy inferred that thorium itself was radioactive and that its activity was constant and did not decay with the time.

The rays from the 23 times precipitated thorium were next sent through a magnetic field and found to be nondeviable; hence it was concluded that thorium emits α-rays. On the other hand, the rays emitted by thorium X and by the active deposit were found to be both deviable and nondeviable.

As a result of the afore-described experiments with thorium, its emanation and its active deposit, Rutherford and Soddy were led to state their theory of radioactive transformations in the *Philosophical Magazine* for September and November 1902, in two papers in the same journal for April 1903, and in one paper in May 1903. These papers were epochal and iconoclastic in character. They changed the whole trend of thought in regard to the atom. The discovery that electrons could penetrate solid substances showed that atoms were not impenetrable, and the brilliant series of papers by Rutherford and Soddy just mentioned showed that atoms were not indestructible. Rutherford and Soddy showed that the results of their experiments on the radioactivity of uranium,

thorium and radium could be explained on three assumptions: (i) radioactivity (that is, the power to impart conductivity to air) is an *accompaniment* of change from one chemical atom to another, (ii) these changes must be occurring within the atom, and (iii) the radioactive elements must be undergoing spontaneous transformation, during which radiations are emitted. Radioactivity, so Rutherford and Soddy said, may therefore be considered as a manifestation of subatomic chemical change. The word "subatomic" strikes one forcibly. Was the idea of the nuclear atom beginning to take shape in the minds of these two brilliant men in 1902? It was not until 1911 that Rutherford put forward the idea of the nuclear atom to explain large angle scattering of α-rays.

Rutherford's last paper in the period to which I have limited myself (to the end of 1904) was communicated on September 16, 1904 to the International Electrical Congress[6] at St. Louis. The paper was published in the *Philosophical Magazine* for November 1904. The Curies had reported in 1903 that a body exposed to radium emanation did not, after removal, completely lose all its activity. A residual activity of about 1/20,000 of the initial activity remained. Rutherford exposed a platinum plate to radium emanation for seven days. The plate was removed from the emanation and placed immediately in the test chamber. For some hours the activity decayed exponentially with a half-life of 28 min. After three days the activity was reduced to 1/300,000 of its initial value. From then on the activity increased linearly with the time up to nine months. But a distinction must be made between the α-ray and β-ray activities. The former increases with the time while the latter remains constant over nine months. The platinum plate lost most of its α-ray activity when heated to 1000°C. At a higher temperature it lost its β-ray activity. Hence, Rutherford argued, the active deposit of slow change consists of two radioactive substances.

The slow-change deposit dissolves in sulfuric acid. If a polished disk of bismuth is put into the solution for several hours, the active matter deposited on the bismuth emits α-rays but no β-rays. It was found that the α-ray activity increases with the time only when the β-ray activity is present.

In the Bakerian lecture delivered in May 1904, Rutherford had recognized the existence of radium A, B and C in the active deposit of rapid change. In September 1904 he added radium D and E in the active deposit of slow change. And so at the end of 1904 Rutherford gave the radium radioactive series as:

Element	Ra	RaEm	RaA	RaB	RaC	RaD	RaE
Rays	α	α	α	none	α, β, γ	β	α
Half-life	—	4 day	3 min	21 min	28 min	40 yr	1 yr

The half-lives of RaD and RaE were estimates based on assumptions that are questionable in 1946, but they were the best estimates that could be made in 1904.

7. Heating Effect

In 1901 Rutherford and McClung made an estimate of the energy of the α-rays radiated into the air from a thin layer of active matter by determining the total number of ions produced by the complete absorption of the α-rays. They found that the energy required to produce a pair of ions in air was 1.90×10^{-10} erg. For 1 gm of uranium spread over a plate in a thin layer they estimated an energy emission of 1.34×10^{6} erg yr^{-1}. Since the activity of pure radium chloride is 1.5×10^{6} times that of uranium, the corresponding rate of emission of energy from radium chloride is 2×10^{12} erg yr^{-1} gm^{-1}, or 2.3×10^{8} erg hr^{-1} gm^{-1}. Even in a thin layer of uranium salt the thickness is still great enough to absorb most of the α-rays, so the value of 2.3×10^{8} erg hr^{-1} gm^{-1} is an underestimate. Converted into heat units, the energy emission from radium is greater than 5.5 cal hr^{-1} gm^{-1}. It is a striking fact that, although the results of Rutherford and McClung's experiments seem to us now to call for an experiment to measure the heat emission of radium directly, an interval of two years passed before Curie and Laborde reported the heating effect of radium in the *Comptes rendus* for March 16, 1903.

In one method the difference of temperature was observed by means of an iron-constantan thermocouple between a tube containing radiferous barium chloride of activity about 1/6 that of pure radium and an exactly similar tube con-

[6] Held in connection with the World's Fair at St. Louis.

taining 1 gm of pure barium chloride. The difference of temperature was 1.5°C. In order to measure the rate of emission of heat, a coil of wire of known resistance was placed in the pure barium chloride, and the strength of the electric current required to raise the barium to the same temperature as the radiferous barium was observed. Curie and Laborde deduced that 1 gm of pure radium (in equilibrium with its transformation products) emits heat at the rate of about 100 cal hr^{-1}. So 1 gm of radium emits per day nearly as much energy as is required to dissociate 1 gm of water.

In later experiments P. Curie found that the rate of heat emission depended on the age of the radium preparation. This pointed to the conclusion that the phenomenon of heat emission of radium was connected with the radioactivity of the element. Rutherford and Barnes in February 1904 reported that the heat emission per gram of radium bromide is 65 cal hr^{-1}, corresponding to an emission per gram of metallic radium of 110 cal hr^{-1}. The emanation occluded in the radium bromide was removed by heating. The emanation was collected in a **U** tube immersed in liquid air. The **U** tube was sealed off, and the heating effects of the "de-emanated" radium and of the emanation tube were then determined at intervals. It was found that, in the course of a few hours after removal of the emanation, the heating effect of the radium fell to a minimum, corresponding to about 30 percent of the original heat emission, and then gradually increased again, reaching its original value after an interval of about a month. The heating effect of the emanation tube was found to increase for the first few hours after separation, to reach a maximum, and then to decay exponentially with time, falling to half its maximum value in about 4 days.

8. Public Interest

A count of the items on radioactivity appearing in the various indexes of *Nature* for the seven-year period ending on October 31, 1902, yields about 60 items. A similar count for the next two-year period gives a total of about 260 items. This increase is indicative of the tremendous stir produced by Rutherford and Soddy's theory of atomic disintegration and by P. Curie and Laborde's discovery that radium continuously maintains itself at a temperature above that of its surroundings.

As was to be expected, the furor spilled over into the popular magazines and newspapers. *Harper's Magazine*, *Atlantic Monthly*, *McClure's* and the *Review of Reviews* all carried articles on radium and radioactivity in 1903. The following items appeared in the St. Louis newspapers:

St. Louis Republic *August 23, 1903*

MOST MYSTERIOUS SUBSTANCE IN NATURE—RADIUM

Experiments made in Paris by discoverers, Monsieur and Madame Curie

St. Louis Republic *August 24, 1903*

BLIND CHILD WAS ENABLED TO SEE

Remarkable experiments with radium and x-ray give scientists high hopes for future

Dead optic nerve revivified

St. Louis Post-Dispatch *September 20, 1903*

RADIUM A THIEF OF FORCE, SAYS KELVIN

British scientist combats theory that it is miraculous generator of energy

[Kelvin thought it impossible that a store of heat, such as P. Curie calculated must be present in a grain of radium, could be lost by this amount of radium in the specified time. Therefore, the only other theory possible was that the energy "was somehow supplied from without."]

St. Louis Post-Dispatch *October 4, 1903*

PRICELESS MYSTERIOUS RADIUM WILL BE EXHIBITED IN ST. LOUIS

A grain of the most wonderful and mysterious metal in the world to be shown in St. Louis in 1904

Its power will be inconceivable. By means of the metal all the arsenals in the world might be destroyed. It could make war impossible by exhausting all the accumulated explosives in the world. . . . It is even possible that an instrument might be invented which at the touch of a key would blow up the whole earth and bring about the end of the world.

It was stated in the *St. Louis Post-Dispatch* for September 24, 1903, that the entire world supply of pure radium was four grains (about 260 mg).

9. Range of α-Rays

During the period 1899 through 1903 it was assumed that the fundamental law of absorption for α-, β- and γ-rays was an exponential one.

Any departure from this law was explained as caused by lack of homogeneity in the rays. However, Mme. Curie had pointed out that for α-rays the experimental values of the intensity fell away more quickly with the thickness of absorbing material than the exponential law required. This fact led to the absurdity that as greater thicknesses of absorbing material were penetrated the rays which remained became less and less penetrating than the original rays. Some curves even indicated a fall to zero at a finite thickness of the absorbing material rather than an approach to zero at an infinite thickness. But the exponential law was a sacred concept, and a crystallization center was needed for a new idea.

The new idea was described by W. H. Bragg in his presidential address at the Dunedin, New Zealand meeting of the Australasian Association for the Advancement of Science, in January 1904. By this time it was known that the α-rays were positively charged particles with a mass twice or perhaps four times that of a hydrogen atom, while the β-rays were electrons with a mass about 1/1000 that of a hydrogen atom. Bragg therefore surmised that the absorption law for the α-rays might be quite different from that for β-rays. Both kinds of rays suffer a continual diminution in speed through the expenditure of energy on ionization; but in the case of the α-rays this is the only cause to which their so-called "absorption" is due. It is clear, said Bragg, that the β-rays are liable to deflection through close encounters with the electrons of the atoms. If with the aid of collimating stops a jet of electrons is projected into air, the jet soon becomes ill-defined and we have exponential "absorption" in the original direction of the jet. However, the α-rays, because of their large mass, do not suffer appreciable deflection and so proceed until the continual loss of energy in ionizing the atoms of the gas reduces their speed to a point where the α-rays cease to ionize. There should thus be a definite range for α-rays from a layer of radioactive material.

Bragg returned to his laboratory at Adelaide, Australia and, working with Kleeman, tested his ideas after obtaining some radium bromide from the other side of the world. The results were published in the *Philosophical Magazine* for December 1904. The apparatus used differed from that described in 1899 by Rutherford for measuring the absorption of α-rays in air by having a shallow ionization chamber, 0.2 cm deep, with its lower plate replaced by wire gauze, and by having two collimating lead stops above a thick layer of radium bromide. Instead of obtaining an exponential curve as Rutherford had done, Bragg obtained a graph which consisted of a succession of straight lines with the angles between consecutive lines rounded off. This rounding was caused by the small distance of the wire gauze from the upper plate of the ionization chamber. The ranges of the α-rays were measured by the distances at which the angles occurred on the graph. Bragg found evidence for four different ranges. These ranges were ascribed by Bragg to the α-rays from radium and from the three disintegration products which emit α-rays and which had been described a few months before by Rutherford.

As indicative of the concepts held at the end of 1904, I quote from Bragg's paper:

> It [the α-ray] is of course far more likely than the β-ray to ionize an atom through which it is passing, because it contains some thousands of electrons and ionizing collision is so much more probable. But a collision between an electron of the flying atom of the α-ray and an electron of the atom traversed can have very little effect on the motion of the α atom as a whole. All the electrons of the α-ray which do not go very close to electrons of the stationary atom are practically undeflected; and probably even one which does is steadied by its connections with its fellows.

At the end of 1904 the concept of the nuclear atom had not yet crystallized in the mind of any physicist.

10. Partly Biographical

Space does not permit me to write all I would like about the five principal workers who took part in unraveling the mystery of radioactivity up to the end of 1904. However, each of these five workers was the conventional type of scientist, differing in that respect from Roentgen, the discoverer of x-rays. I shall therefore content myself with mentioning only certain salient facts and commenting on them. I am adding W. H. Bragg to the list because I knew him personally and for other reasons given below.

Henri Becquerel.—Born in Paris on December 15, 1852; died on August 25, 1908. He was 43 when he obtained the first indications of what two years later came to be known as radioactivity. From his early work on uranium and its compounds, Becquerel concluded that the penetrating and persistent radiations were a property of the element uranium.

Pierre Curie.—Born in Paris on May 15, 1859; died on April 19, 1906 as the result of a traffic accident in Paris. He was 39 when he wrote his first paper on radioactivity. In 1880 the brothers, J. and P. Curie, devised an electric balance in which a charge on an electroscope can be balanced by a charge due to the piezoelectric effect of quartz. The principle of this balance was used by Pierre and Marie Curie in their measurements on radioactivity. In 1895, Pierre Curie published his justly famous work on the *Magnetic properties of bodies at different temperatures.*

Marie Sklodowska Curie.—Born in Warsaw on November 7, 1867; died on July 4, 1934. The Curies were married in 1895. Mme. Curie was 31 when she wrote her first paper on radioactivity (the discovery that thorium was radioactive). The guiding principle in Mme. Curie's early work was that radioactivity is an atomic process.

Ernest Rutherford.—Born near Nelson in the South Island of New Zealand on August 30, 1871; died on October 13, 1937. He was a pupil of J. J. Thomson's before being appointed professor of physics at McGill University, Montreal, Canada. All of his work described in this article was done at Montreal. He was 27 when he wrote his first paper on radioactivity and 31 when he and Soddy began writing their epochal series of papers on radioactive transformations.

Frederick Soddy.—Born at Eastbourne, England, on September 2, 1877. He was a demonstrator (instructor) in chemistry at McGill University when he and Rutherford wrote their epochal series of papers. The first of this series was published when Soddy was 25 years old.

William Henry Bragg.—Born in Cumberland, England, on July 2, 1862; died on March 12, 1942. At the suggestion of J. J. Thomson he applied for and obtained the chair of physics and mathematics at the University of Adelaide, Australia, when he was 24 years old. It is often said that, if a physicist does not make his mark before he is 40, he never will. Bragg is a brilliant exception to this rule. Bragg was an unknown physicist at an out-of-the-way antipodean university when he was 40. He had as yet published no papers in physics. His paper on the range of α-rays appeared when he was 42.

Nobel Prizes were awarded for work in radioactivity as follows: Becquerel and Pierre and Marie Curie jointly in 1903; Rutherford in 1908; Marie Curie in 1911; and Soddy in 1921.

11. Elements and Atoms

In the period 1896 through 1904 there were four distinct advances in the ideas of the world of physics. In chronological order these ideas were: (i) radioactivity is a property of the element uranium; (ii) it is a property of the atoms of a radioactive substance; (iii) it is a property of a certain fraction of the atoms of a radioactive substance over a short interval of time; and (iv) the atoms that emit α- or β-radiations undergo transformation into atoms with chemical and radioactive properties different from those of the parent atoms.

From the vantage point of 1946, we know that radioactivity cannot properly be described as a property of an element. For instance, P^{32} is radioactive while P^{31} is not, although both are isotopes of the chemical element phosphorus. But, going back to 1896, we find that, although Dalton's atomic theory of chemistry was generally accepted, the physical atom as we know it in 1946 was a vague concept. According to Dampier, in *A history of science* (1944), Ostwald, the physical chemist, proposed to discard atomic conceptions in favor of energetics and thermodynamics shortly before the new physics began to use atomic ideas in an extreme form. During the period 1896 through 1904, we note the emergence of the physical atom as distinct from the vague and theoretical chemical atom. However, the proof that atoms exist in a physical sense was obtained at the cost of surrendering the concept that atoms are indestructible and immutable. Radioactivity showed that certain atoms can undergo spontaneous change.

SCIENTISTS believe that in the making of the pattern of Nature there is a gradual perfection. In art there appears to be no progress; who dares say that sculpture today has advanced beyond Scopas? In art there is only change; science, which traces its beginnings back to that identical moment when art began—the charging bison on the cave walls in Spain—is conscious of growth.
—DONALD CULROSS PEATTIE, *Green Laurels.*

Generalized Impedance Function for the Laplace Equation

CHARLES KITTEL
Massachusetts Institute of Technology, Cambridge 39, Massachusetts

THE introduction of an impedance function has been a powerful tool in the solution of electromagnetic[1–3] and acoustic[4,5] problems. In these fields the method is often known as the hyperbolic-tangent method. The purpose of the present paper is to show how the method can be extended to solutions of the Laplace equation such as arise in static problems in electric and thermal conduction, and electric and magnetic field distribution. The principal advantage of the explicit introduction of the new function occurs when we wish to determine, in electrical terminology, the input impedance of a network. The general philosophy behind the use of an impedance function can be applied to all boundary-value problems, if the function is defined in a suitable way.

Acoustic Impedance

Let us consider in some detail the use of the acoustic impedance function.[4] The specific acoustic impedance $Z(x)$ is defined as the ratio between excess pressure p and particle velocity u at the point x for the frequency $\omega/2\pi$. For a wave made up by superposing two plane waves traveling in opposite directions we have

$$p = P_+ e^{ik(x-ct)} + P_- e^{-ik(x+ct)}, \tag{1}$$

$$\begin{aligned} u &= (i\rho kc)^{-1}\partial p/\partial x \\ &= (\rho c)^{-1}[P_+ e^{ik(x-ct)} - P_- e^{-ik(x+ct)}], \end{aligned} \tag{2}$$

from which one establishes

$$Z(x) = p/u = \rho c \tanh(\psi + i\omega x/c), \tag{3}$$

where the ratio P_-/P_+ has been written as $-e^{-2\psi}$.

Now the boundary conditions at the interface between two mediums of different acoustic properties are that the pressure and the particle velocity must be continuous across the interface. If p and u are continuous, then $Z(x)$ is also continuous. This gives us an important property of the impedance function: the *specific acoustic impedance is continuous across the interface between two different mediums.*

In many important problems which arise in practice we are interested only in the reaction of a "network" on the driving source. Here we can deal with impedances throughout and get a solution by the application of only one-half the number of boundary conditions that we would have to consider if we were required to determine the pressure and the velocity all along the network. We start with the assigned value of the terminal impedance of the network and adjust ψ in Eq. (3) so that $Z(L)$ at the termination is equal to the terminal impedance Z_T. Then we work back to the input end. At each discontinuity of the characteristic acoustic impedance we determine a new ψ by requiring continuity of Z across the discontinuity. Finally we arrive at the input section and get the input impedance $Z(O)$ of the network.

Applying this procedure to the simple acoustic network shown in Fig. (1) we get

$$Z(L) = (\rho c)_2 \tanh(\psi_2 + i\omega L/c_2) = Z_T, \tag{4}$$

which determines ψ_2. We go on to determine ψ_1 from the boundary condition at $x=a$:

$$\begin{aligned} &(\rho c)_1 \tanh(\psi_1 + i\omega\alpha/c_1) \\ &\qquad = (\rho c)_2 \tanh(\psi_2 + i\omega a/c_2). \end{aligned} \tag{5}$$

The input impedance of the network is given by

$$Z(O) = (\rho c)_1 \tanh \psi_1, \tag{6}$$

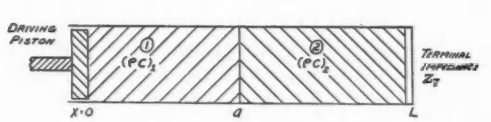


FIG. 1. Acoustic network.

[1] S. A. Schelkunoff, *Electromagnetic waves* (Van Nostrand, New York, 1943).
[2] J. C. Slater, *Microwave transmission* (McGraw-Hill, New York, 1942).
[3] J. A. Stratton, *Electromagnetic theory* (McGraw-Hill, New York, 1941).
[4] For further details consult P. M. Morse, *Vibration and sound* (McGraw-Hill, New York, 1936).
[5] L. Foldy and H. Primakoff, *J. Acous. Soc. Am.* **17**, 109 (1945).

where Eqs. (4) and (5) suffice to determine ψ_1 in terms of Z_T and the constants of the network. If now we specify the driving force $F[=pA]$ on the input piston, we can calculate the velocity amplitude of the piston from $u=p/Z(0)$ if the mechanical inertia of the piston is neglected.

Extension to the Laplace Equation

We are now in a position to consider the application of the impedance concept to static boundary value problems. Suppose that the solution U of the Laplace equation is expressed as a linear combination,

$$U=Au_1+Bu_2, \tag{7}$$

of two independent solutions u_1 and u_2. This is equivalent to superposing waves traveling in opposite directions. A parallel situation to the acoustic problem previously discussed is the following:

(*a*) On the $N-1$ surfaces which divide the region under consideration into N subregions, we have the continuity conditions,

$$c_iU_i=c_{i+1}U_{i+1}, \tag{8}$$

$$k_i\partial U_i/\partial n=k_{i+1}\partial U_{i+1}/\partial n, \tag{9}$$

where n is the normal to the surface separating subregions i and $i+1$.

(*b*) In the "terminal," or Nth, subregion, the ratio B_N/A_N is preassigned. This is equivalent to specifying the terminal impedance.

(*c*) In the "input," or first, subregion, B_1 (or A_1) is preassigned. This is equivalent to specifying the driving force.

To get a complete solution to the problem it is necessary to determine the $2N$ coefficients (A's and B's) by solving a set of $2N$ linear algebraic equations.

As before, we are often interested only in the A_1 associated with the preassigned B_1. That is, we want to know the quantity somewhat analogous to an *input impedance*. Here we have only to solve a set of N linear equations in the N ratios

$$Q_i=B_i/A_i \tag{10}$$

to determine Q_1; then A_1 is found as B_1/Q_1. The Q's are analogous to the ψ's of the acoustic case.

The reduction of the problem from $2N$ to about N equations leads to substantial simplification when the problem is complicated. If we wish to determine A_n or B_n, however, we have to solve $n-1$ equations in addition to the $N+1$ needed to determine A_1. To determine the terminal A_N or B_N we must in any case solve the full set of $2N$ equations.

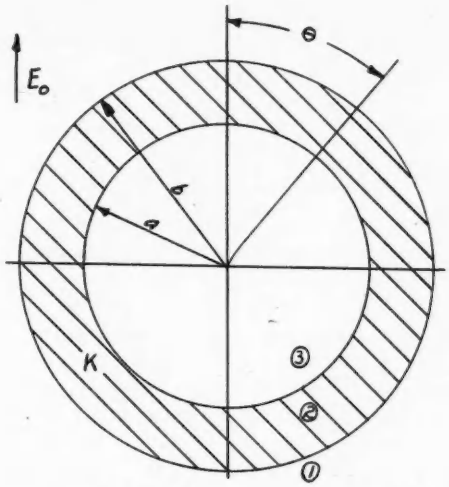


FIG. 2. Spherical dielectric shell in a uniform field.

If we are going to solve for the Q's, it is convenient to deal throughout with the function

$$Z=(k/c)(\partial U/U\partial n)$$
$$=(k/c)(u_1'+Qu_2')/(u_1+Qu_2), \tag{11}$$

where we have written u_1' for $\partial u_1/\partial n$ and u_2' for $\partial u_2/\partial n$. By virtue of Eqs. (8) and (9) this function is continuous across all boundaries. We shall call Z the *generalized impedance function* by analogy with the impedance as defined in the acoustic problem.

Let us consider a particular problem as an example. It is desired to determine the electric field distribution outside a spherical dielectric shell, of inner radius a and outer radius b, placed in a uniform electric field E_0 (Fig. 2). The potential is of the form

$$V=Ar^{-2}P_1(\cos\theta)+BrP_1(\cos\theta), \tag{12}$$

and the generalized impedance function is

$$Z=\frac{\epsilon}{r}\left\{\frac{-2+Qr^3}{1+Qr^3}\right\}, \tag{13}$$

where ϵ is the dielectric constant. The terminal Q_3 is infinite because A_3 must be zero. We have

$$Z_3 = 1/r. \tag{14}$$

Now at $r=a$ we must have Z continuous:

$$\frac{\epsilon}{a}\left\{\frac{-2+Q_2a^3}{1+Q_2a^3}\right\} = \frac{1}{a}. \tag{15}$$

This determines Q_2. Similarly at $r=b$,

$$\frac{1}{b}\left\{\frac{-2+Q_1b^3}{1+Q_1b^3}\right\} = \frac{\epsilon}{b}\left\{\frac{-2+Q_2b^3}{1+Q_2b^3}\right\}, \tag{16}$$

which determines Q_1. Now $-B_1$ is just the applied field ("driving pressure") E_0; from Q_1 and B_1 we have $A_1 = -E_0/Q_1$.

It is of interest to consider the forms assumed by the generalized impedance function for the Laplace equation in some special coordinate systems. When terms of more than one type of angular dependence are present, each type should be considered separately, as is usually done in these problems, and an impedance function should be defined for each type of angular dependence.

Spherical coordinates:

$$U = (r^{-n-1} + Qr^n)P_n(\cos\theta), \tag{17}$$

$$Z = r^{-1}\left[\frac{-(n+1)+nQr^{2n+1}}{1+Qr^{2n+1}}\right]. \tag{18}$$

Cylindrical coordinates:

$$U = (r^{-n} + Qr^n)\cos n\theta, \tag{19}$$

$$Z = \frac{n}{r}\left[\frac{-1+Qr^{2n}}{1+Qr^{2n}}\right]. \tag{20}$$

Rectangular coordinates:

$$U = (e^{+ny} + Qe^{-ny})\cos nx, \tag{21}$$

$$Z = n\left[\frac{1-Qe^{-2ny}}{1+Qe^{-2ny}}\right] = n\tanh(ny+q), \tag{22}$$

where $Q = e^{-2q}$.

The multiplying factor k/c has been omitted from the foregoing expressions for Z. In representative types of problems k/c is defined as follows:

Type	k/c
Electric potential	ϵ, the dielectric constant
Magnetic vector potential	$1/\mu$, where μ is the permeability
Temperature	k, the thermal conductivity
Electric current	$1/\sigma$, where σ is the electric conductivity

The constant k/c can be regarded as analogous to the *characteristic impedance* of the medium in electromagnetic radiation and acoustic problems.

This treatment can be generalized to other boundary conditions. If the conditions are $F_1(U)$, $F_2(U)$ continuous, then the generalized impedance function can be taken as $Z = F_1/F_2$.

GREATER even than the greatest discovery is it to keep open the way to future discovery. This can only be done when the investigator freely dares, moved by an inner propulsion, to attack problems not because they give promise of immediate value to the human race, but because they make an irresistible appeal by reason of an inner beauty. . . . In short, there should be in research work a cultural character, an artistic quality, elements that give to painting, music and poetry their high place in the life of man.—JOHN JACOB ABEL.

Two Elementary Experiments to Demonstrate the Photoelectric Law and Measure the Planck Constant

AUSTIN J. O'LEARY
The City College, New York 10, New York

IN each of the researches that have been carried out to test the Einstein photoelectric law,

$$\tfrac{1}{2}mv_{\max}^2 = h\nu - w, \qquad (1)$$

the method has been to apply to the anode of a photoelectric cell a negative potential relative to the cathode and to measure, for different frequencies of monochromatic light, the minimum potential V necessary to reduce the photoelectric current to zero. For any given frequency, we have

$$\tfrac{1}{2}mv_{\max}^2 = (V+V')e, \qquad (2)$$

where V' is the contact potential difference from anode to cathode, and e is the electronic charge. Combining Eqs. (1) and (2), one gets

$$h\nu = Ve + K, \qquad (3)$$

where K is a constant equal to $V'e+w$. The difficulties involved in an accurate determination of the cut-off points of the current-potential curves make this research method unsuited to student use.

A greatly simplified method is here described. It employs an electrostatic system in which the blocking potential is automatically generated by the action of the light. It has the great advantage that V is directly measurable. There are two variations of the method. The first is designed for the use of beginners; the second, for those students to whom a string electrometer may be entrusted.

Condenser-Galvanometer Method

The arrangement is indicated in Fig. 1. With the rocker key K closed on the "charge" side and monochromatic light incident on the cathode of the photo-cell, electrons will continue to reach the anode until the condenser acquires the potential difference V, Eq. (2). The potential difference existing at any given time is measured by discharging the condenser through the galvanometer G; the measuring system is calibrated by means of a standard cell. By changing the single-pole double-throw switch S from its normal operating position *1* to position *2*, one gets, from the galvanometer reading, a relative measure of the maximum emission from the cathode for the different frequencies of light used and a relative measure of the light intensity for different intensities of the same frequency. I shall first present the results obtained and shall then discuss the condenser and photo-cell that were used.

There are two basic facts to be demonstrated:

(1) *It is to be shown that, for any given frequency, V is independent of the light intensity.* The strong mercury ultraviolet line, 3650A, is probably most satisfactory for this purpose since the filter which isolates it blocks out all longer wave-lengths so that the experiment may be conducted under ordinary illumination. At different intensities of illumination, obtained by placing the mercury arc at different distances from the cell, the charge acquired by the condenser is measured as a function of the

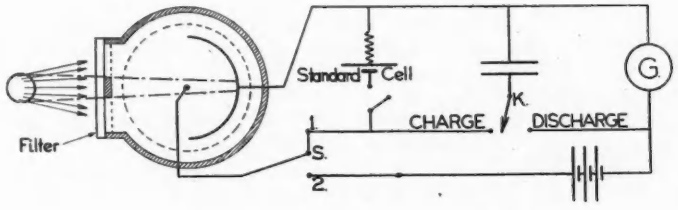


FIG. 1. Schematic arrangement of condenser, galvanometer and photo-cell for laboratory experiment to demonstrate the photoelectric law.

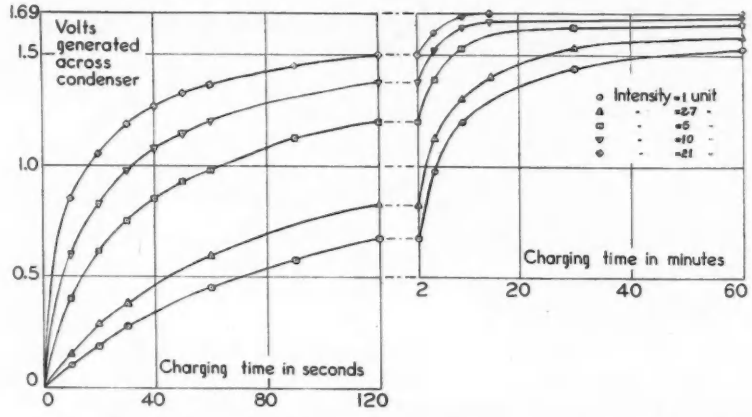


FIG. 2. Graphs showing the potential difference across the condenser as a function of the time allowed for it to acquire charge as a result of the photoelectric emission produced by five different intensities of the mercury line, 3650A.

allowed charging time. The graphs in Fig. 2 illustrate a set of results in which the light intensity was varied to the extent of twenty-one-fold. As the intensity is decreased, the condenser acquires charge more slowly; but all the curves approach the same maximum value of V within the limits imposed by leakage and by "reverse current" due to photoelectric emission from the anode (induced, in this experiment, by reflected light).

(2) *It is to be shown that, for a given photo-cell, V is a linear function of the frequency ν; and taking the function to be given by Eq. (3), h is determined.* Having demonstrated the first of the foregoing facts, one can legitimately carry out this second part of the experiment at the maximum intensity available. The curves in Fig. 2, along with a measure of the cathode emission, give a rough idea of the charging time that should be allowed in each case. The results are listed in Table I and are plotted in Fig. 3.

TABLE I. Measured values of the blocking potential difference V for five different frequencies of light.

Source	Filter	λ(A)	ν (10^{14} sec^{-1})	V(v)
Mercury arc	Corning 586 (8 mm)	3650	8.22	1.69
	Wratten 50	4358	6.89	1.20
	Wratten 62	5461	5.49	0.68
	Wratten 22	5770	5.20	0.57
		5791	5.19	
Sodium arc or flame	Corning 348	5890	5.09	0.53
		5896	5.09	

From Fig. 3 it may be seen that the relationship between V and ν is strictly linear. The slope of the graph is h/e and is seen to be 3.71×10^{-15} v sec. From this we get $h=3.71\times10^{-15}$ ev sec, or, after multiplying by 1.60×10^{-12} to change electron volts to ergs,

$$h=5.94\times10^{-27} \text{ erg sec.}$$

This value of h is about 9 percent below the most probable value as determined by photoelectric methods up to 1932[1] and about 10 percent below Birge's 1941 value.[2] This is what one would expect with apparatus in which some reverse current and some leakage occur. The condenser stops charging when the charging current is reduced to a value just equal to the combined reverse and leakage currents. Since the latter two currents are at least roughly proportional to V, one would expect a linear relationship between ν and the measured values of V but would expect the slope h/e of the graph to be less than the true value. However, in view of the almost complete lack of experiments that can be performed in an elementary laboratory to measure fundamental constants, the student should realize what an accomplishment it is to measure h with an error no larger than 10 per-

[1] A. L. Hughes and L. A. DuBridge, *Photoelectric phenomena* (McGraw-Hill, 1932), p. 26; $h=6.542\times10^{-27}$ erg sec.

[2] R. T. Birge, *Rev. Mod. Physics* **13**, 233 (1941); $h=\{2\pi^2c^3F^5/R_\infty N_0^5(e/m)\}^{\frac{1}{3}}=(6.624\pm0.002)\times10^{-27}$ erg sec.

cent, particularly if he reads about the many difficulties that had to be overcome and the special research equipment needed in the original determination of facts and the measurement of the Planck constant.[3]

Condenser and photo-cell.—For the values of V in Table I, a 0.5-μf condenser gives a satisfactory deflection on discharge through a Leeds and Northrup wall galvanometer. Of all the condensers tried, the Western Electric 227-A was found to have the highest leakage resistance.

An RCA-935 photo-cell was used. The straight-wire anode of this cell is sealed in at the top to provide improved insulation between anode and cathode through a maximum path of glass. As a precaution against leakage along the outer surface of the glass resulting from adsorbed water vapor, the glass, with the exception of a window, was coated with a thin film of ceresin. Whatever leakage occurred across the cell was probably caused by an invisibly thin film of alkali metal deposited on the inner surface of the glass during the "flashing" process. Except in humid summer weather, the RC value for the condenser plus photo-cell was found to be of the order of 250,000 ohm farads, representing a leakage resistance[4] of the order of 500,000 megohms.

Leakage is not the main source of error. It is reverse current, much more than leakage current, that limits the possibilities of the experiment. To have to shield the anode from direct light is a nuisance in an elementary experiment. It involves use of a narrow cylindrical source; it reduces the intensity appreciably below what one might otherwise have; and even when the anode is carefully shielded from direct light, it is still not protected from reflected and scattered light. A photo-cell with negligible anode emission would improve the experiment enormously. Besides the obvious advantages of increased accuracy and no bother with shielding, such a cell could be used in two additional ways as follows:

(i) It could be calibrated by means of the mercury lines and then be used to measure the short wave-length transmission limit of filters. In particular, this could be done quickly and conveniently by using the cell in conjunction with a string electrometer.

(ii) It could be used in an instructive experiment without need for a mercury arc. One could use the light from a continuous spectrum source along with readily procurable filters having a sharp cut-off on the short wave-length side. Present cells cannot be used in these two ways owing to the fact that all wave-lengths shorter than the threshold value contribute toward reverse current but not toward charging current.

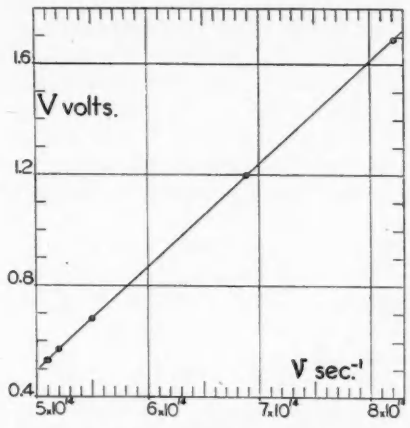


FIG. 3. Graph showing the linear relationship between the values of V and ν in Table I.

If the anode of a cell were made in the form of a loop instead of a single wire, it should be possible to outgas it during the evacuation process and, after flashing, to evaporate the sensitizing material that happens to be deposited on it; the anode could be heated momentarily by an electric current without injuring the cathode surface. Whatever photoelectric emission there might be from the basic anode material would be negligibly small in comparison with that from the sensitized cathode, which is not the case when the anode itself is sensitized.

The glass envelope of the RCA-935 cell is designed for high transmission of the ultraviolet down to about 2000A, and this cell is considerably more expensive than the ordinary commercial cell. A less costly glass might be used in a cell designed for this experiment; it would not need to transmit wave-lengths shorter than 3650A.

String Electrometer Method

In this arrangement, the condenser, galvanometer, and rocker key are replaced by a string electrometer and the standard cell by a direct-reading potentiometer; the sensitivity of the electrometer varies somewhat in the course of time, so it is best to determine the potential corresponding to any given deflection of the fiber by observing the potentiometer setting that gives the same deflection at the time a reading is taken.

The photo-cell was mounted in an inverted position directly on the electrometer, with the

[3] Reference 1, pp. 1–32.

[4] Of course, the resistance R of a condenser depends to a large extent on how it is measured and on the immediate past history of the charge on the condenser. The above value of R represents the leakage resistance of the system under normal operating conditions as nearly as it has any meaning.

anode connected to the fiber and the cathode to the grounded case. To insure the best possible insulation for the anode-fiber combination, external connection was effected by means of a camera cable release which could be pushed into a tiny spring socket attached to the fiber binding post; when released, it springs back, leaving an air gap for insulation. The capacitance of the system is extremely small. Hence, the time required for it to acquire the full potential difference V is negligible even when the light intensity is small. However, if the light is shut off completely, the system quickly comes to ground potential, its RC value being relatively small; in this arrangement, practically all the leakage occurs across the inner surface of the glass envelope of the photo-cell.

The reading of the electrometer for a given frequency does not change as the light source is moved a considerable distance away from the cell, and so it may be shown in a matter of seconds that V is independent of the light intensity. Since the deflection corresponding to each frequency occurs instantaneously, a complete observation of the relation between V and ν may be carried out in a few minutes. Results in agreement with those in Table I and Fig. 3 were obtained, with this one difference: it was observed that each value of V was 0.1 to 0.2 v less when the light was confined to the lower part of the cathode surface (that is, a portion of the surface nearest the anode cap, since the tube is mounted in an inverted position) than when it was confined to the upper part. Thus, the work function for the lower part of the surface of the cell used must be 0.1 to 0.2 ev larger than for the upper part of the surface;[5] no doubt, this is caused by a difference in the thickness of the alkali deposit. So it appears that, when the whole cathode surface is used, the effective value of the work function is a composite of different values which vary continuously from one portion of the surface to another.

During the humid summer months, it was necessary to employ a drying agent. A side-arm containing phosphorous pentoxide was sealed into the housing for the photo-cell and fiber binding post, and the whole enclosure was made gastight. The measurements were then unaffected by even the highest humidity. It did not seem feasible to provide a similar enclosure for the condenser-galvanometer method; hence the latter could be used to good effect only in a dry atmosphere.

I wish to thank Professor B. Kurrelmeyer of Brooklyn College for lending me his string electrometer for a considerable period of time, and Messrs. G. Kayser and C. E. Ostlund for constructing various arrangements of the apparatus.

[5] I do not wish to commit myself as to a more exact value for this difference until really accurate measurements can be made with a tube having negligible anode emission.

CHEMISTRY has been termed by the physicist the messy part of physics, but that is no reason why the physicists should be permitted to make a mess of chemistry when they invade it.—FREDERICK SODDY.

Brachistochrone and Tautochrone Curves for Rolling Bodies

ERIC RODGERS
University of Alabama, University, Alabama

THE brachistochrone and tautochrone properties of the cycloid for bodies sliding without friction are well known. The purpose of the present article is to consider brachistochrone and tautochrone curves for bodies *rolling* without frictional loss of energy. It will be shown that for rolling motion, as for sliding motion, the brachistochrone and tautochrone curves are such that the center of mass of the body moves on a cycloid. In the following discussion, a sphere is chosen as the rolling body.

Brachistochrone

Here the problem is that of finding the equation of the curve along which a homogeneous sphere will roll such that its center of mass will move from O to A (Fig. 1) in the least time if no energy is lost because of friction.

Choose axes as shown in Fig. 1. When the center of mass of the sphere has descended a vertical distance y, the energy equation is

$$Mgy=\tfrac{1}{2}Mv^2+\tfrac{1}{2}I\omega^2,$$

where v is the velocity of the center of mass, ω is the angular velocity and I is the moment of inertia of the sphere about an axis through its center of mass.

In Fig. 1 let ρ be the radius of curvature of the curve on which the center of mass of the sphere moves. Since this curve is parallel to the curve on which the ball rolls, the corresponding radius of curvature of the latter is $\rho+r$, where r is the radius of the ball. The linear velocities of the center of mass and the point of contact of the ball are, respectively, $\rho d\alpha/dt$ and $(\rho+r)d\alpha/dt$.

If the ball were rolling on a straight line, its angular velocity would be its linear velocity along the line divided by its radius. Here the ball is rolling on a curve and this value must be diminished by the angular velocity of the radius of curvature of the curve to get the true angular velocity.[1] Therefore

$$\omega=\left(\frac{\rho+r}{r}\right)\frac{d\alpha}{dt}-\frac{d\alpha}{dt}=\frac{\rho}{r}\frac{d\alpha}{dt}=\frac{v}{r}.$$

This illustrates a general principle, namely, that as a ball rolls on any curve, its angular velocity is the linear velocity of its center divided by its radius.

The equation of energy now becomes

$$Mgy=\tfrac{1}{2}Mv^2+\tfrac{1}{2}(\tfrac{2}{5}Mr^2)v^2/r^2,$$

whence $v=(10gy/7)^{\frac{1}{2}}$. But, since $v=ds/dt$, the time of descent is given by the equation

$$t=(7/10g)^{\frac{1}{2}}\int_0^a ds/y^{\frac{1}{2}}.$$

The problem is now to find the equation of the curve along which the time integral is a minimum. This means that the integral

$$\int_0^a \frac{(1+y'^2)^{\frac{1}{2}}}{y^{\frac{1}{2}}}dx \qquad (1)$$

is to be made a minimum.

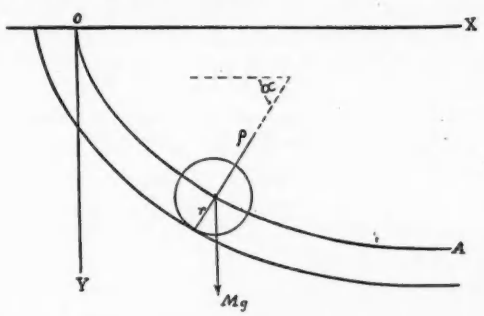


FIG. 1. Brachistochrone and tautochrone curves for rolling ball.

[1] To see that this is true, consider Fig. 1. Suppose that, at a given instant, the ball should leave the curve and roll along the line tangent to the curve at the point of departure. After rolling a certain distance on the tangent line, suppose the ball should stop rolling and the tangent line then be rolled on the curve until the ball is again on the curve. The net angle through which the ball would roll would then be the angle through which it rolled on the tangent line minus the angle through which the tangent line would have to roll to put the ball back on the curve. But the angle between the tangent lines at the two points is the same as the angle between the corresponding radii of curvature of the curve,

The integral (1) is of the form

$$\int_0^a F(x, y, y')dx,$$

the value of which depends on the path of the curve joining O and A. For the integral to be a minimum, it is necessary that the Euler condition of the calculus of variations be satisfied. By this condition the equation

$$\frac{\partial F}{\partial y} - \frac{\mathrm{d}(\partial F/\partial y')}{\mathrm{d}x} = 0$$

must be satisfied for each point on the curve. When the Euler condition is applied to the integrand of (1) the resulting differential equation is

$$1 + \frac{2y}{1+y'^2}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0.$$

Integration of this differential equation gives

$$x = -(2cy - y^2)^{\frac{1}{2}} + c\cos^{-1}\frac{c-y}{c} + c' \qquad (2)$$

as the equation of the curve of quickest descent for the center of mass of the rolling ball. Since the curve passes through the origin, the constant c' is zero.

Equation (2) is that of a cycloid, the radius of whose generating circle is c. It may be put in the convenient parametric form

$$x = c\theta - c\sin\theta, \quad y = c - c\cos\theta. \qquad (3)$$

The brachistochrone curve on which the ball rolls is a curve parallel to the cycloid, but is not itself a cycloid. From Eqs. (3) and Fig. 1, the equations of the brachistochrone are easily found to be

$$\begin{aligned} x &= c\theta - c\sin\theta - r\cos\tfrac{1}{2}\theta, \\ y &= c - c\cos\theta + r\sin\tfrac{1}{2}\theta. \end{aligned} \qquad (4)$$

It can be shown that each point on this curve has the same slope and the same center of curvature as the corresponding point on the cycloid.

Tautochrone

It will now be shown that the curve of Eqs. (4) is also a tautochrone for a rolling ball of radius r. This means that the time taken for the ball to roll from any initial point to the lowest point of the curve is independent of the position on the curve of the initial point.

Let x_0, y_0 be the coordinates of the center of mass of the ball in its initial position. The equation of energy is now

$$Mg(y - y_0) = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2,$$

or

$$g(y - y_0) = \frac{7}{10}v^2.$$

This gives

$$t = \left(\frac{7}{10g}\right)^{\frac{1}{2}} \int_{x_0}^{a} \left(\frac{1+y'^2}{y-y_0}\right)^{\frac{1}{2}} \mathrm{d}x, \qquad (5)$$

where a is the abscissa of the lowest point. The proof of the tautochrone property will first be given directly by showing that this integral for the time is independent of the choice of the point (x_0, y_0) on the cycloid.

When a ball of radius r rolls on the curve of Eqs. (4), its center of mass moves on the cycloid of Eqs. (3). Consequently

$$\mathrm{d}y/\mathrm{d}x = \cot\tfrac{1}{2}\theta$$

and

$$(1+y'^2)^{\frac{1}{2}}\mathrm{d}x = 2c\sin\tfrac{1}{2}\theta\,\mathrm{d}\theta.$$

Substitution of these values in Eq. (5) gives

$$t = \left(\frac{7c}{10g}\right)^{\frac{1}{2}} \int_{\theta_0}^{\pi} \frac{2\sin\frac{1}{2}\theta}{(\cos\theta_0 - \cos\theta)^{\frac{1}{2}}} d\theta,$$

where θ_0 corresponds to the point (x_0, y_0).

The integrand of this integral is infinite at the lower limit θ_0. To get around this difficulty, this limit will be changed to θ_0'. After integration the limit of the result will be found as $\theta_0' \to \theta_0$. The time integral now becomes

$$t = \operatorname*{Lim}_{\theta_0' \to \theta_0} \left(\frac{7c}{10g}\right)^{\frac{1}{2}} \int_{\theta_0'}^{\pi} \frac{2\sin\frac{1}{2}\theta}{(2\cos^2\frac{1}{2}\theta_0 - 2\cos^2\frac{1}{2}\theta)^{\frac{1}{2}}} d\theta$$

$$= \operatorname{Lim}_{\theta_0' \to \theta_0} \left(\frac{7c}{5g}\right)^{\frac{1}{2}} \int_{\theta_0'}^{\pi} \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta_0 \left[1 - \left(\frac{\cos \frac{1}{2}\theta}{\cos \frac{1}{2}\theta_0}\right)^2\right]^{\frac{1}{2}}} d\theta$$

$$= \operatorname{Lim}_{\theta_0' \to \theta_0} \left(\frac{7c}{5g}\right)^{\frac{1}{2}} \left[2 \cos^{-1} \frac{\cos \frac{1}{2}\theta}{\cos \frac{1}{2}\theta_0}\right]_{\theta_0'}^{\pi},$$

or

$$t = (7c/5g)^{\frac{1}{2}}\pi. \tag{6}$$

This value of the time is constant and thus independent of the initial position on the curve, which proves that the curve of Eqs. (4) is a tautochrone curve.

It may now be shown that the curve of Eqs. (4) is a tautochrone only for a ball of radius r, or that it is the tautochrone only of a ball whose center of mass moves on the cycloid. If r in Eqs. (4) be considered as an arbitrary parameter that can have any value, the equations represent an infinite family of parallel curves. The cycloid is the particular curve of the family for which r is zero.

Consider a ball of any radius R rolling on any curve of the family. The center of mass of the ball will then move on a curve of the family, the equations of which may be written as

$$\begin{aligned} x &= c\theta - c \sin \theta - K \cos \tfrac{1}{2}\theta, \\ y &= c - c \cos \theta + K \sin \tfrac{1}{2}\theta, \end{aligned} \tag{7}$$

where K is the distance of this curve from the cycloid. The gravitational torque on the ball is

$$\begin{aligned} L &= MgR \sin (\tan^{-1} dy/dx) \\ &= MgR \cos \tfrac{1}{2}\theta. \end{aligned}$$

As the ball rolls up its curve, the angle through which it rotates is the arc length of the curve of Eqs. (7) divided by R. This arc length, measured from the lowest point, is

$$\begin{aligned} S &= \int_{\theta}^{\pi} \csc \tfrac{1}{2}\theta (c - c \cos \theta + \tfrac{1}{2}K \sin \tfrac{1}{2}\theta) d\theta \\ &= 4c \cos \tfrac{1}{2}\theta + \tfrac{1}{2}K(\pi - \theta). \end{aligned}$$

The angle ϕ through which the ball rolls is therefore

$$\phi = (4c/R) \cos \tfrac{1}{2}\theta + (K/2R)(\pi - \theta).$$

It is seen that L is not proportional to ϕ unless K is zero. This means that oscillations would not be angular harmonic and the curve is not a tautochrone unless K is zero, and if K is zero the center of mass moves on the cycloid.

So, if K is zero, the ball rolls on the tautochrone, and the time of descent given by Eq. (6) can be obtained very simply by means of the relation

$$T = 2\pi \left(\frac{I}{L/\phi}\right)^{\frac{1}{2}}.$$

Discussion

Owing to the smallness of rolling frictional force, apparatus can be constructed that allows the principles to be demonstrated adequately. It was the successful operation of such an apparatus constructed in our laboratory that suggested the problem to the writer. The apparatus was made by cutting a cycloid curve out of a wide 2-in. board. A smooth groove was then cut in the board along the curve to a constant depth of $\frac{1}{8}$ in., making the bottom of this groove a tautochrone and brachistochrone curve for a rolling ball of diameter $\frac{1}{4}$ in.

For demonstration of the brachistochrone property, some other curve cut in another board is placed beside the brachistochrone curve. A vertical piece of board is arranged to stop balls rolling on both curves just as they reach the bottom. As balls rolling down the two curves strike the stop, the clicks are clearly resolved, with the one on the brachistochrone being heard first.

For the tautochrone property, two balls are released simultaneously on the tautochrone curve, one on each side of the stop. Regardless of the relative heights from which the two balls are released, only one click is heard as the balls strike the stop.

An alternative method for demonstrating the tautochrone property is to remove the stop and observe the time for a number of complete oscillations of the ball. The time of descent from any height is given by Eq. (6). For our apparatus the radius of the generating circle is 18.38 cm. The computed time of descent of a $\frac{1}{4}$-in. ball is 0.5090 sec. With such a ball the average time of descent for 20 trials was 0.5093 ± 0.0002 sec.

As long as the center of mass of the ball descends on a curve that is close to the cycloid, the time of descent differs very little from that on the cycloid. This makes it unnecessary in a demonstration to use a ball of exactly the right size. As an example, an experimental test using a 1-in. ball with our apparatus gave the value 0.5028 ± 0.0002 sec as the average of 20 trials. The time is less for the larger ball because its center of mass moves on a shorter curve.

The writer is grateful to Dr. B. A. Wooten, who constructed the demonstration apparatus which suggested the problem, and who offered valuable suggestions throughout the study.

Some Demonstrations in Mechanics

MARIO IONA, JR.
*The University of Chicago, Chicago 37, Illinois**

IN second-year college physics it is often desirable to demonstrate some of the more complicated relations of physical phenomena. Even though the analytic treatment of the experiments under discussion may not be beyond the grasp of the student, a demonstration of the significance of the various terms is generally a valuable addition. The phenomena discussed in this paper are frequently mentioned in textbooks but not discussed sufficiently to make them meaningful to the student. These demonstrations will be of interest in a freshman mechanics course and of value in clarifying the more analytic discussion in an intermediate course.

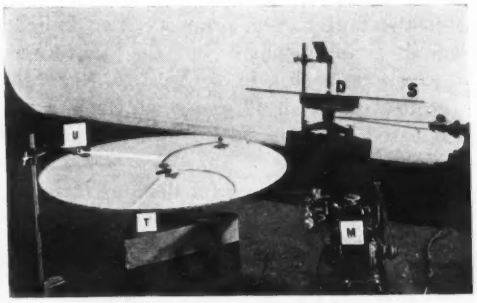


FIG. 1. Demonstration of the curving of the path of a ball rolling over a rotating glass disk due to Coriolis force. The ball moves in a straight line as can be seen by observing the shadow projection.

* Now at the University of Denver, Denver 10, Colorado.

Simple Cases of Coriolis Forces

The subject of Coriolis forces is frequently mentioned in elementary courses; but it is difficult for the student to follow any abstract treatment. Some of the effects of Coriolis forces, however, such as the motion of the Foucault pendulum or the behavior of winds, are sufficiently important to justify a discussion. Fortunately, a number of these important cases can be presented without a general treatment. However, more correlations between the various phenomena may be pointed out, such as the Coriolis force experienced by a boy on a turntable when he pulls masses held with his stretched arms toward the axis.

In a discussion of the Coriolis force it is important to show clearly the difference between the observations of an observer in a rotating system and those of another observer in an inertial system—a system that is at rest or moving with constant velocity. A demonstration of this difference consists in showing that the path of an object which is not under the action of any force follows a straight path in an inertial system, while an observer in a rotating system will observe a curved path. If the observer in the rotating system still wishes to apply Newton's laws of motion, he will have to postulate that there is a force, namely the Coriolis force,

acting on the object to deflect it from a straight path.

In the experiment illustrated by Fig. 1, the path of a ball may be observed in the two coordinate systems. The ball marks its path on the rotating glass plate over which it is rolling, and its path in the stationary system is obtained by observing the shadow of the ball as it moves in the direction of the light beam.

The steel ball C, which has been dipped into India ink, acquires an initial speed by rolling down the U-shaped chute U (whose reflection is also on the glass disk). By changing the height and position of the chute it is possible to let the ball move across the plate with any desired velocity. In Fig. 1, the ball had been released so as to make initial contact with the plate with a velocity directed along the radius (for an outside observer) coming from the center of the glass disk. The glass disk is covered with white paper on its lower surface so as to make the India ink tracks appear more clearly. The whole glass disk is mounted on a turntable T which is moving counter-clockwise with a constant angular speed of about $\frac{1}{2}$ rev/sec. The light beam from a projection lantern coming from the left foreground casts a shadow S of the glass disk on the screen seen in the background and makes possible the observation of the shadow of the ball, which stays directly below the shadow of the chute near D. The curved path shown at A was obtained in an earlier experiment where the ball made initial contact with the plate at a point that was moving in the same direction as the ball, as indicated by the arrow.

Although there is, of course, some friction between the glass plate and the wet steel ball, the resulting sidewise deflection is small compared to the one due to the Coriolis force. If the experiment is performed so that the ball comes from the center of the glass plate, the deviation due to friction as seen in the shadow projection is opposite to the deviation from the straight path due to the Coriolis force as seen on the rotating plate.

Steiner's Theorem

In elementary textbooks Steiner's theorem is frequently stated without any attempt at proof or explanation. The analytic proof that the moment of inertia of an object about a certain axis is equal to the sum of the moments of inertia MR_0^2 which the object would have if all the mass were concentrated at its center of mass plus the moment of inertia I_0 of the object about a parallel axis through the center of mass can be given if the students are accustomed to considering the mass as consisting of mass elements. It seems desirable, however, to demonstrate the significance of the center of mass in this connection. The analytic proof shows, of course, that certain simplifications are possible only if the reference axis passes through the center of mass. A demonstration of the physical relations, however, will be of greater value for the beginning student.

To have an object for which an approximate calculation in terms of point masses is most plausible, a dumbbell with unequal masses, m_1 and m_2, is chosen for illustration. Figure 2 shows a top view of the system. The dumbbell can rotate about an axis perpendicular to the plane of the paper through point C, and that in turn is mounted on a turntable with a vertical axis through A. The masses can be adjusted so that the axis through C passes through the center of mass of the dumbbell. Provisions are made to arrest axis C in order to prevent rotation of the dumbbell about C if one wishes.

It can easily be shown algebraically that if axis C passes through the center of mass of the dumbbell, any rotation about axis A will not cause any torque about axis C; thus the dumbbell, if it is at rest initially, will keep its orienta-

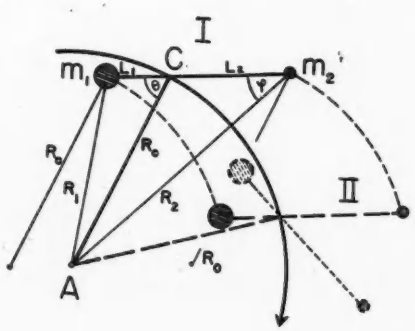


FIG. 2. Demonstration of Steiner's theorem. The dumbbell m_1m_2, which is free to rotate about C, will not change its orientation as it moves about axis A.

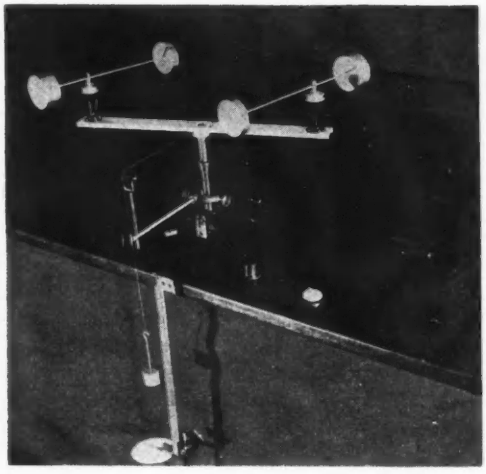


FIG. 3. Apparatus for the demonstration of Steiner's theorem.

tion in space as it swings in an arc around A from position I to position II. (The torques with respect to axis C due to the centripetal forces necessary to keep both masses moving on circles with radius R_0 are equal in magnitude and opposite in sign.) Furthermore, any rotation of the dumbbell about axis C through the center of mass of the dumbbell will not exert any torque about axis A, since the centripetal forces for rotation about C are equal in magnitude and opposite in sign. This shows that the rotations about A and C are entirely independent. The motion can also be analyzed from a consideration of the moment of inertia of the dumbbell about axis A; and one finds that if C is at the center of mass of the dumbbell, the moment of inertia about A is independent of the angle θ.

If rotations about axes A and C occur simultaneously with the same angular velocity and acceleration, the moment of inertia will be the sum of the moments of inertia for the two independent motions. Such a motion will result if the dumbbell is prevented from rotating freely about axis C so that it will move as a rigid body from position I into the dotted position II.

In the apparatus used (Fig. 3) the masses are cylindrical rather than spherical and, for better balance, the whole system is symmetrical with respect to the main axis A. The cylindrical masses can be moved along the common rod and can be clamped at any distance from the axis C. By means of pins the bicycle bearings on these axes can be prevented from turning.

With the apparatus indicated, the following experiments can be performed:

(1) Direct measurement of the moment of inertia of the dumbbell about axis C and axis A by suspending weights from strings wrapped around the pulleys on these axes and observing the time of descent through a certain distance. In these measurements it may be necessary to take friction into account by making measurements with more than one weight. If the dumbbell is free to turn about axis C and if this axis passes through the center of mass of the dumbbell, the moment of inertia of the dumbbell for rotation about A will be that of the total mass, m_1+m_2, at the distance R_0.

(2) Measurement of the increase of the moment of inertia about axis A, if C passes through the center of mass when axis C is arrested so that no free motion takes place about this axis. In this case the increase will be found to correspond to the second term in Steiner's theorem, namely, the moment of inertia I_0 of the dumbbell about the axis through C, as determined in experiment (1).

(3) Observation of the mutual effects of rotations about the two axes if axis C passes through the center of mass of the dumbbell. No motion about axis A will result from a spinning of the dumbbell about axis C, and rotation about axis A will not affect the orientation of the dumbbell with respect to the room.

(4) Observation of the effect of rotation about the two axes if C does *not* pass through the center of mass of the dumbbell. In this case spinning the dumbbell about axis C will result in an oscillating torque about axis A so that angular momentum about this axis may be conserved. If the system is rotating about axis A, the dumbbell will adjust itself about axis C in such a manner that the center of mass is as far from axis A as possible, since the torques of the centripetal forces about A do not balance in this case. In the experiment, care must be taken to prevent coupled oscillations of the two dumbbells in order to demonstrate the tendency for

rotation about the axis of largest moment of inertia without disturbance.

The experiment for measuring the increase of the moment of inertia by arresting axis C, which amounts to about 30 percent, can be improved by suspending the whole system as a torsion pendulum, as shown in Fig. 4.

The actual measurements with the arrangement shown in Fig. 3 were rather satisfactory, considering the rough bicycle bearings used. The corrections arising from the fact that the masses are not point masses amounts to 2 percent. The discrepancy between the moment of inertia about axis C and the increase of moment of inertia due to arresting axis C was 7 percent in the turntable experiment and 3 percent in the experiment on the torsion rod. No corrections have been applied for the transfer of energy to the dumbbell due to friction when the dumbbell was free to rotate about axis C. If the rotation about axis A was sufficiently fast (5 to 10 rad/sec) the dumbbell turned only $\frac{1}{4}$ rev with respect to the room during 8 rev of the turntable.

A Generalization of the Physical Pendulum

Most college physics textbooks introduce the physical, or compound, pendulum as an example of a rotational oscillator and call attention to the property that there are several points of suspension for which the pendulum has the same period. Demonstration of this property usually consists of showing a Kater pendulum or a meter stick supported at various points along its length. However, in restricting the examples to essentially one-dimensional objects an interesting generalization is overlooked.

An equation that allows one to arrive in a convenient way at the relations between the period of a physical pendulum and the point of suspension is the following expression for the period:

$$T=2\pi[(k_0^2+h^2)/gh]^{\frac{1}{2}}.$$

In this equation Steiner's theorem was used to express the moment of inertia with respect to the axis of suspension in terms of h, the distance from the center of mass to the axis, and k_0, the radius of gyration with respect to a parallel axis through the center of mass.

If the foregoing equation is compared with that for the simple pendulum of length l, namely,

$$T_0=2\pi(l/g)^{\frac{1}{2}},$$

one can determine the value of h for which the period of the physical pendulum is equal to that of the simple pendulum. Equating the periods leads to the quadratic equation $h^2-lh+k_0^2=0$, with the familiar conditions for the two roots, h_1 and h_2,

$$h_1+h_2=l \quad \text{and} \quad h_1h_2=k_0^2.$$

These relations are usually discussed only for the four points that lie on a line passing through the center of mass. The generalization which is ignored is that the period is the same for any suspension at distances h_1 or h_2 from the center of mass, independent of the position of the axis as long as its orientation with respect to the pendulum does not change, that is, as long as k_0 is constant.

An impressive illustration of this fact is offered by a set of four meter sticks suspended, respectively, at the end, at the 66.7-cm mark, and, by means of strings, at points 16.7 cm and 50.0 cm from the center, as shown in Fig. 5. All these

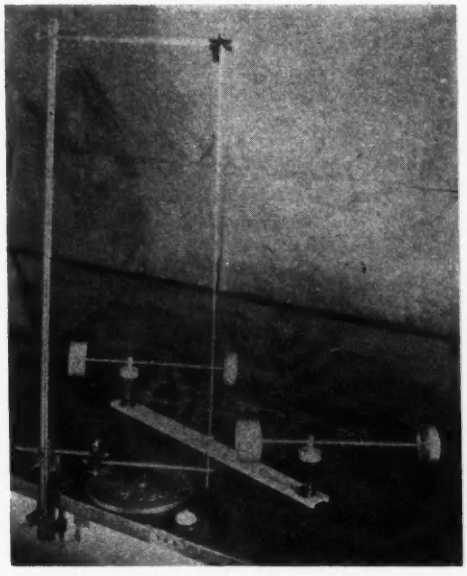


FIG. 4. Torsion pendulum for the demonstration of Steiner's theorem.

forms of the physical pendulum will have the same period as a simple pendulum of length 66.7 cm, which may be used for comparison.

Another method of demonstrating this dependence on distance only is to use as a physical pendulum a triangular plate with holes for suspension on a pin S, as indicated in Fig. 6. The holes are drilled along two concentric circular arcs about the center of mass C, the circles having radii h_1 and h_2, respectively.

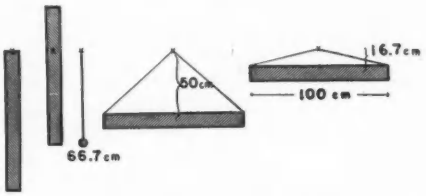


FIG. 5. Meter sticks suspended at 16.7 or 50 cm from the center of mass have the same period.

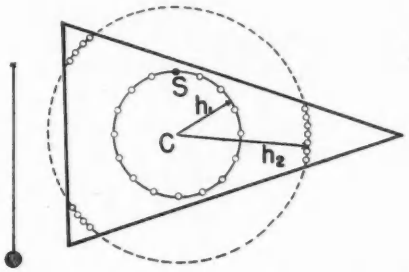


FIG. 6. The locus of all points of suspension for which a physical pendulum (triangular plate) has a given period around axes of fixed orientation consists of two circles around the center of mass.

These demonstrations have instructional value as applications of the pendulum relations and as illustrations of Steiner's theorem. Furthermore, they stimulate the student's interest because the equations predict simple, verifiable results which have probably not been anticipated.

Grateful appreciation is expressed to Mr. J. P. Karbler, Ryerson Laboratory demonstrator, who generously cooperated in building and setting up the apparatus for these experiments, and to Professor F. L. Verwiebe, of Hamilton College, whose advice was always available.

IN its own field, the science of mechanics has been so successful and so exact that it has been extended to other sciences verbally, but not factually. It was not the mechanistic physicists who made the mechanistic philosophy "not only reigning, but without a rival." It was the biologists, who have pictured life as a mechanical evolution of matter; the psychologists, who have identified sensation, thought and the soul with mechanical energy; the sociologists, who have tried to substitute rigorous social laws for individual responsibility and free will; and who have degraded Christian ideals of other-worldliness to a humanitarian hedonism. It is these scientists and monistic metaphysicians, who have ruined modern philosophy.—LOUIS T. MORE, *Sci. Mo.* **56**, 504 (1943).

Special Circuits for the Measurement of Resistance and Inductance

D. S. AINSLIE
University of Toronto, Toronto 5, Canada

IN carrying out accurate measurements of inductance and resistance by means of special bridge circuits, considerable time is wasted unless the approximate values of the constants are known before starting the work. To overcome this difficulty the two circuits described in this paper were devised and have proven very useful in laboratory work. For student use, these circuits possess an important advantage over apparatus specially manufactured for work of this nature, in that they are simple enough for the student easily to understand the principles embodied in their use.

The first circuit (Fig. 1) was designed for the measurement of the resistance of an electrolyte by a substitution method. It consists essentially of a step-down transformer T, ratio 5 to 1, with the secondary connected across a potentiometer rheostat consisting of two resistance units of 30 and 400 ohms, respectively, connected in series.

The test circuit connects to the potentiometer at the contacts C_1 and C_2 and consists essentially of three fixed resistors in series, a double-pole double-throw switch S_3, and three pairs of terminals R, V and X. For the pair of terminals X, flexible leads terminating in Mueller clip connectors were employed. The values of the fixed resistors, which were controlled by switches S_1 and S_2, were chosen so as to be of the same order of magnitude as the unknown resistances for which measurements were to be made. The values of 300, 1000 and 10,000 ohms used in this unit proved satisfactory for unknown resistances ranging from 200 to 8000 ohms. The other components of the circuit—a noninductive resistance box, a high-resistance, rectifier-type alternating current voltmeter, and a cell with electrolyte—were connected to the terminals R, V and X, respectively.

To carry out a measurement the instrument was connected to a 110-v, 60-c/sec a.c. line and the switch S_3 was thrown so as to connect the unknown resistance into the circuit. By means of the contacts C_1 and C_2 the applied potential difference was varied until a full-scale reading was obtained on one of the low range scales available on the voltmeter. The switch S_3 was now thrown over so as to substitute the resistance box in the circuit in place of the unknown, and its resistance was adjusted until the voltmeter reading was the same as for the first test. The value of the unknown resistance is equal to that of the resistance box. The highest sensitivity was secured when the fixed resistance, as controlled by switches S_1 and S_2, was of the same order of magnitude as the unknown. The accuracy of the measurements was satisfactory for the purpose in view; the difference between

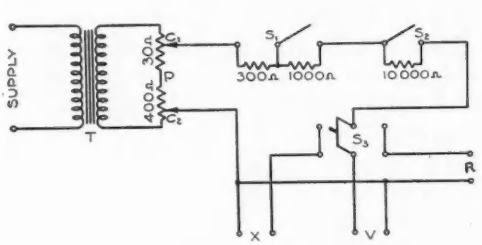


FIG. 1. Circuit for the measurement of resistance: T, step-down transformer; P, potentiometer resistors; S_1, SPDT knife switch; S_2, SPST knife switch; S_3, DPDT knife switch: V, R, X, terminals.

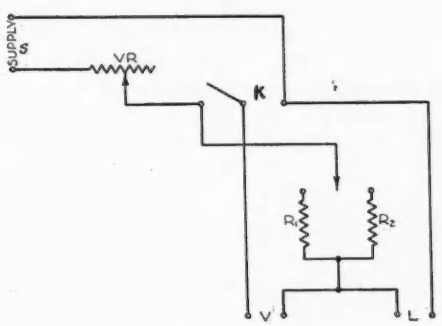


FIG. 2. Circuit for the measurement of inductance: VR, variable rheostat, 0–200 ohms; R_1 and R_2, 200- and 1000-ohm wire-wound precision resistors; K, SPDT knife switch; S, V, L, terminals.

these preliminary measurements and those secured on a slide-wire bridge instrument operated on a 1000-c/sec a.c. supply line was usually less than 4 percent. One instrument of this type enabled ten pairs of students working on slide-wire bridge experiments to make the desired preliminary measurements.

TABLE I. Values of k for L in millihenries and a frequency f of 2000 c/sec.

Potential difference V_1	k for comparison resistance R, 200 ohms	R, 1000 ohms
0.4	40	200
.8	20	100
1.6	10	50

Figure 2 is a wiring diagram of the second circuit, which was designed for the measurement of inductance. The essential components are a 2000-ohm variable rheostat (radio type), VR, two wire-wound precision-type resistance units R_1 and R_2, a single-pole double-throw switch K, and three pairs of terminals, S, V and L. A clip lead was provided in the circuit so that either R_1 or R_2 could be used therein.

The circuit was connected at S to an oscillator, and the unknown inductance and a multirange voltmeter were connected to the terminals L and V, respectively. To determine the value of the inductance, measurements were made of the potential differences V_1 and V_2 for the coil and the resistance unit, respectively. The variable resistance VR was employed to keep these potential differences within the voltmeter range. A frequency f of 2000 c/sec was employed so as to render the potential difference V_2 practically independent of the resistance of the coil. Under these conditions the following equation is very nearly correct:

$$V_2 = i \cdot 2\pi f L,$$

where $i[=V_1/R]$ is the current in the circuit, and R is the resistance of the noninductive resistance unit in series with L. When this equation is solved for L, expressed in millihenries, we have

$$L = (1000R/2\pi f V_1) V_2 = k V_2.$$

The value of V_1 may be selected so as to make k an integral number within the limits of accuracy of the other measurements. The value of L is then determined with a minimum of calculation. Typical values of these constants are listed in Table I. These values may be represented by the general equation $V_1 kf/R = 160$. A few determinations of self-inductance by means of this circuit are given in Table II along with the corresponding values as determined by a bridge method.

TABLE II. Values of self-inductance L.

Coil	Resistance R (ohms)	Potential difference (v) V_1 for resistance R	V_2 for coil L	Factor k	Self-inductance of coil L (mh) L, R circuit	Bridge
Standard 10-mh unit	200	1.6	1.03	10	10.3	10
		0.8	0.51	20	10.2	
			0.505		10.1	
					Av. 10.2	
Inductance coil No. 1	200	1.6	4.1	10	41	
	1000	1.6	0.80	50	40	
					Av. 40.5	40
Coil No. 2	200	1.6	9.80	10	98	96
	1000	1.6	1.92	50	96	
					Av. 97.0	
Coil No. 3	1000	1.6	4.80	4.80	240	244
			4.76		238	
			4.96		248	
					Av. 242.0	

For satisfactory measurements with this method it is essential that the wave form of the a.c. source conform closely to that of a sine curve and that an a.c. voltmeter of good quality be employed for the measurements of potential difference. The values listed in Table II were obtained with the help of a Clough Brengle beat-frequency oscillator as a source of alternating emf and a Weston analyser instrument, Model No. 772, as a voltmeter.

I HAVE no economic radar to penetrate the future, but we can make it what we will it to be. Of that I am sure.—BERNARD BARUCH.

Refractions by a Thick Lens Which Is Equivalent to a Compound Lens System

MASON E. HUFFORD
Indiana University, Bloomington, Indiana

I

A COMPOUND lens comprises several coaxial elements of glass in air. Evidently two curved surfaces can always be found that will cause the same refraction of the incident and emergent rays as is imposed by the combined action of the several surfaces of the compound system. This is the same as saying that a thick lens can always be found which has the same focal lengths as the compound system, and therefore the focal lengths, cardinal points and the thickness of the thick lens can be regarded as belonging to the compound lens.

In many of the analyses of a thick lens the type of lens equation and the presence of principal points and principal planes of unit magnification are assumed, and a geometry is set up on these assumptions.[1] It seems, however, a much more direct procedure to apply the laws of refraction at the two surfaces of the thick lens, using paraxial rays, and to derive the thick-lens formula as a result of such refraction. This method also establishes the principal planes of unit magnification and the Helmholtz relation as natural consequences of the bending of the light.

Suppose that light originating at O (Fig. 1) traverses a distance p, strikes a surface S_1 and is then brought to a focus in the glass at a point I whose distance from S_1 is q. The radius of curvature of S_1 is R_1, and the angles of incidence and refraction are i and r, respectively. The foregoing quantities pertaining to the surface are related by the equations

$$\frac{1}{p}+\frac{n}{q}=\frac{n-1}{R_1}=\frac{1}{f_1}, \tag{1}$$

where n is the refractive index of the glass and f_1 is the focal length of the refracting surface. In order that the formula may be general, n and f are assumed to be constant for light of all frequencies. The refractive index n is i/r, and the refractive index of air is assumed to be unity. Only paraxial rays are considered so that the cosines of the angles of incidence and refraction are unity.

Now let a second refracting surface S_2 (Fig. 2) be interposed between S_1 and I. This surface produces a real image I' of the virtual object I, at a distance q' from S_2. The relation of the quantities is

$$\frac{1}{q'}-\frac{n}{p'}=\frac{n-1}{R_2}=\frac{1}{f_2}. \tag{2}$$

Thus we have an object O at a distance p from the first surface, and its image I' at a distance q' from the second surface, the space between surfaces being filled with glass. We have made no convention of signs, nor shall we do so.

From Eq. (1),

$$q=npf_1/(p-f_1).$$

Substitution of this value in Eq. (2) gives

$$\frac{1}{q'}-\frac{n}{[npf_1/(p-f_1)]-t}=\frac{1}{f_2}, \tag{3}$$

where t is the axial distance between surfaces.

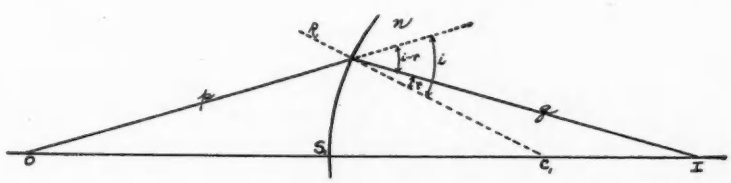


FIG. 1.

[1] Hardy and Perrin, *The principles of optics*, p. 50; Southall, *Mirrors, prisms and lenses* (1923), p. 362; Robertson, *Introduction to physical optics* (ed. 2), p. 100.

From Eq. (3),

$$p\left[\frac{nf_1f_2}{n(f_2+f_1)-t}-\frac{f_2t}{n(f_2+f_1)-t}\right]+q'\left[\frac{nf_1f_2}{n(f_2+f_1)-t}-\frac{f_1t}{n(f_2+f_1)-t}\right]=pq'-\frac{f_1f_2t}{n(f_2+f_1)-t}. \quad (4)$$

Let

$$\frac{nf_1f_2}{n(f_2+f_1)-t}=F, \quad (5) \qquad \frac{f_2t}{n(f_2+f_1)-t}=\beta, \quad (6) \qquad \frac{f_1t}{n(f_2+f_1)-t}=\alpha; \quad (7)$$

then Eq. (4) may be written,

$$pF-p\beta+q'F-q'\alpha=pq'-Ft/n. \quad (8)$$

Now if to each member of Eq. (8) is added $\alpha F+\beta F$, the equation becomes

$$F(p+\alpha)+F(q'+\beta)=pq'+p\beta+q'\alpha-F(t/n)+\alpha F+\beta F. \quad (9)$$

Addition and subtraction of $\alpha\beta$ in the right-hand member gives

$$F(p+\alpha)+F(q'+\beta)=(p+\alpha)(q'+\beta)-F(t/n)+\alpha F+\beta F-\alpha\beta. \quad (10)$$

Now

$$-F(t/n)+\alpha F+\beta F-\alpha\beta=-\frac{f_1f_2t}{n(f_2+f_1)-t}+\frac{nf_1^2f_2t}{[n(f_2+f_1)-t]^2}+\frac{nf_1f_2^2t}{[n(f_2+f_1)-t]^2}-\frac{f_1f_2t^2}{[n(f_2+f_1)-t]^2}$$

$$=\frac{f_1f_2t}{[n(f_2+f_1)-t]^2}[-\{n(f_2+f_1)-t\}+\{n(f_2+f_1-t\}]=0.$$

Hence Eq. (10) comes to

$$\frac{(p+\alpha)+(q'+\beta)}{(p+\alpha)(q'+\beta)}=\frac{1}{F},$$

or

$$\frac{1}{p+\alpha}+\frac{1}{q'+\beta}=\frac{1}{F}, \quad (11)$$

the equation of the thick lens. This can be written in the form,

$$\frac{1}{P}+\frac{1}{Q}=\frac{1}{F}, \quad (12)$$

where P is measured from a point distant α from the first surface and Q is measured from a point distant β from the second surface. The points on the axis distant α and β from the surfaces establish the positions of two planes—the principal planes—perpendicular to the axis, from which the distance of object and distance of image are measured for a thick lens, as these distances are measured from the center of a thin lens.

If a combination of two thin coaxial lenses of focal lengths f_1 and f_2, separated by a distance t in air, receives light from an object distant p from the first lens, as in the case of the Ramsden eyepiece, then Eq. (1) becomes

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f_1},$$

and Eq. (2) becomes

$$\frac{1}{q'}-\frac{1}{p'}=\frac{1}{f_2},$$

where $p'=q-t$. Then a final equation similar to Eq. (12) is easily derived; but now F, α, β are

replaced by

$$F=f_1f_2/(f_2+f_1-t),$$

$$\alpha=f_1t/(f_2+f_1-t),$$

$$\beta=f_2t/(f_2+f_1-t),$$

from which the cardinal points can be determined. Images can be found from the analog of Eq. (11), namely,

$$\frac{1}{p+\alpha}+\frac{1}{q'+\beta}=\frac{1}{F}.$$

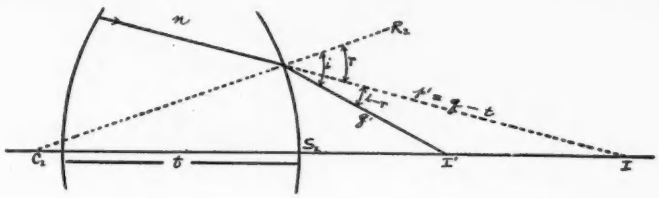


FIG. 2.

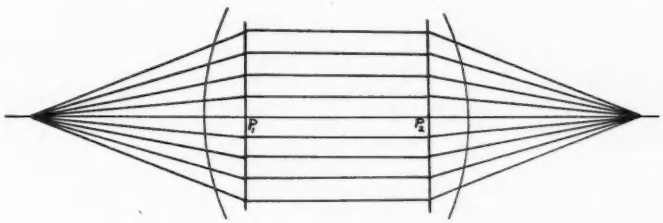


FIG. 3.

If p is less than $F-\alpha$, then that form of the equation which applies to virtual images must be used.

If rays are incident on a thick lens parallel to the axis, then $1/(p+\alpha)=0$, and $q'+\beta=F_2$, the second focal length of the lens. Similarly, when rays emerge parallel to the axis, $p+\alpha=F_1$, the first focal length of the lens. Between the principal planes it is usual to draw the rays parallel to the axis, as in Fig. 3. Of course, this is a convention, for the only rays which, between the principal planes, actually are parallel to the axis are those that originate at or pass through the focal point f_1 and on emergence pass through f_2.

II

For any point in any finite object, incident and emergent rays can always be found that make equal angles with the principal axis. The intersections of such pairs of rays with the axis establish two points, N_1 and N_2 (Fig. 4), called the nodal points.

The intersections of the principal planes with the axis are the principal points, P_1 and P_2, and these, together with the nodal points and the focal points F_1 and F_2, constitute the cardinal points of the thick lens or of an equivalent compound lens. A seventh point, the optical center of the lens, is defined as that point on the axis through which passes a ray whose path on emergence is parallel to its path on incidence.

III

To find the focal length F_1 (say to the left) of the thick lens it is only necessary to find a point on the axis (to the left of the surface S_1), rays from which will emerge from the surface S_2 parallel to the axis. Let surface S_2 (Fig. 5) separate glass on the left from air on the right. From the triangles shown,

$$\frac{1}{q'}+\frac{n}{q}=\frac{1}{f_2}, \tag{13}$$

where f_2 is the focal length of the surface S_2. Now if q' becomes infinite, $q=nf_2$. The value of p, Fig. 6, which will give a virtual image at a distance $q-t$ from S_1 will locate the focal point F_1. From the geometry,

$$\frac{1}{p}-\frac{n}{q-t}=\frac{1}{f_1},$$

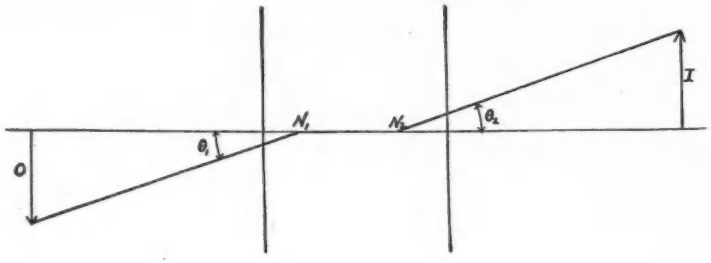


FIG. 4.

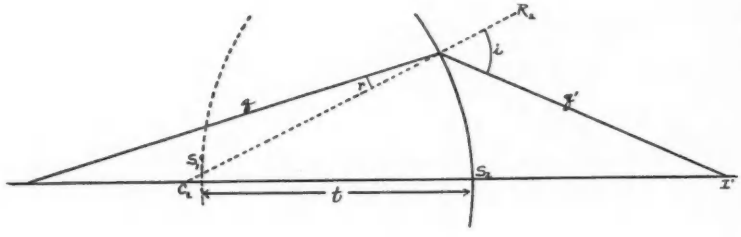


FIG. 5.

where f_1 is the focal length of the surface S_1. Then

$$p=\frac{f_1(nf_2-t)}{n(f_2+f_1)-t}=FS_1;$$

and, since

$$\alpha=\frac{f_1t}{n(f_2+f_1)-t},$$

then

$$p+\alpha=\frac{nf_1f_2}{n(f_2+f_1)-t}=F_1. \tag{13a}$$

It is seen that if the focal length of the thick lens is defined in the usual way as the point, distant F_1 from the first principal plane, where rays parallel to the axis are focused, then we are justified in making the assumption of Eq. (5). Similarly, for a ray entering S_1 (Fig. 7),

$$\frac{n}{q}+\frac{1}{p}=\frac{1}{f_1},$$

and $q=nf_1$.

At the second surface, $q-t$ is the distance from S_2 of a virtual object, an image of which is formed at a distance q' from S_2, so that,

$$\frac{1}{q'}-\frac{n}{q-t}=\frac{1}{f_2},$$

$$q'=\frac{f_2(nf_1-t)}{n(f_2+f_1)-t}=FS_2,$$

$$\beta=\frac{f_2t}{n(f_2+f_1)-t},$$

and

$$q'+\beta=\frac{nf_1f_2}{n(f_2+f_1)-t}=F_2, \tag{13b}$$

the second focal length. It is seen, therefore, that the focal lengths F_1 and F_2 are equal. The first principal focal point is the same distance from the first principal plane that the second principal focal point is from the second principal plane, and

$$\frac{p+\alpha}{q'+\beta}=\frac{F_1}{F_2}=1. \tag{14}$$

If Eq. (6) is multiplied by f_1 and Eq. (7) by f_2, the left-hand members become equal. Hence,

$$f_1\beta=f_2\alpha, \tag{15}$$

and therefore

$$f_2(f_1+\alpha)=f_1(f_2+\beta).$$

Thus we can write the equations

$$\frac{f_1+\alpha}{f_2+\beta}=\frac{f_1}{f_2}=\frac{\alpha}{\beta}=\frac{R_1}{R_2}. \tag{16}$$

Of course, the focal lengths F_1 and F_2 are equal only when the medium of the object space and that of the image space are the same. If the refractive index of the object space is n_1 and that of the image space is n_2, then the focal lengths bear the relation

$$n_1F_1=n_2F_2;$$

the length of a light path in a material medium relative to that in a vacuum is obtained by multiplying the path length in the medium by the refractive index.

IV

In a thin lens the magnification is usually expressed by the ratio y_2/y_1, where y_2 is the length of the image and y_1 is the length of the object; and this ratio is equal to q'/p, the ratio of the distances of image and object from the center of the lens. The analogous expression for a thick lens or an equivalent compound lens is

$$M=y_2/y_1=(q'+\beta)/(p+\alpha).$$

This can be proved as follows.

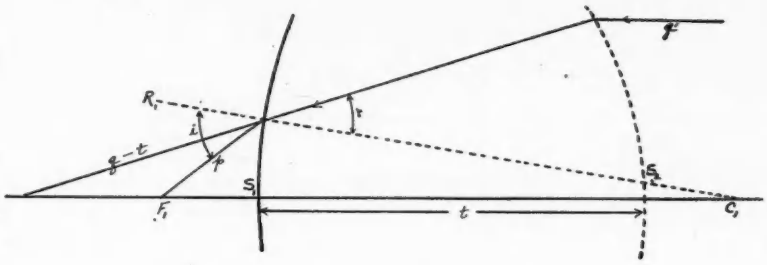


FIG. 6.

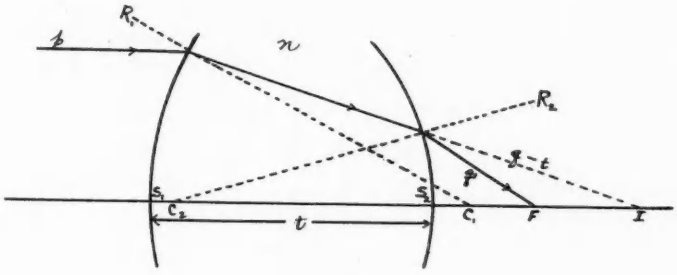


FIG. 7.

A ray from a point in the object and passing through F_1 (Fig. 8) proceeds from the first principal plane parallel to the axis. A ray from the same point in the object and parallel to the axis must on emergence pass through the second focal point F_2. Let the distance between object and F_1 be U, and let the distance between image and F_2 be V. Now from the geometry of the figure, $y_1/y_2=U/F_1$ and $y_2/y_1=V/F_2$. Then,

$$(y_1+y_2)/y_2=(U+F_1)/F_1$$

and

$$(y_2+y_1)/y_1=(V+F_2)/F_2,$$

from which

$$\frac{y_1}{y_2}=\frac{F_2(U+F_1)}{F_1(V+F_2)}=\frac{F_2(p+\alpha)}{F_1(q'+\beta)}.$$

But $F_2=F_1$; hence,

$$\frac{\text{size of image}}{\text{size of object}}=\frac{y_2}{y_1}=\frac{q'+\beta}{p+\alpha}. \qquad (17)$$

When object and image are equal in size they are equally distant from their respective principal planes. From Eq. (11), when $p+\alpha=q'+\beta$, then $2/(p+\alpha)=1/F$, or $2/(q'+\beta)=1/F$. Hence, when image and object are equal,

$$p+\alpha=2F \quad \text{and} \quad q'+\beta=2F.$$

V

The values of α and β are so related that any ray from a point on the axis incident on the first principal plane at a distance P_1M_1 from the axis (Fig. 9) will approach the image from a point in the second principal plane at a distance P_2M_2 from the axis such that

$$P_1M_1=P_2M_2. \qquad (18)$$

This can be proved as follows.

Let a ray from O strike the first surface S_1 at a point a distance A above the axis. The ray produced strikes the first principal plane at M_1, at a distance P_1M_1 above the axis. The ray is refracted along the line q, t; strikes the second surface S_2 at a distance B from the axis and is again refracted along q' to I'. The emergent ray extended back cuts the second principal plane at M_2, distant P_2M_2 from the axis. Now $P_1M_1/A=(p+\alpha)/p$, since the angles are small. Also, $P_2M_2/B=(q'+\beta)/q'$ and $A/B=(q+t)/q$.

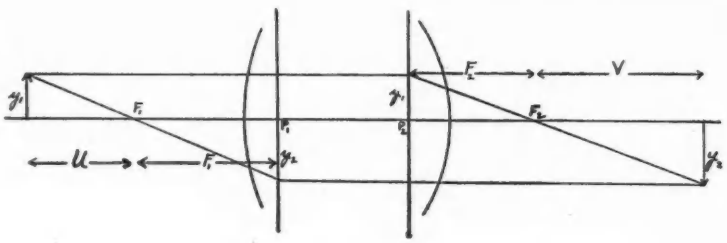


FIG. 8.

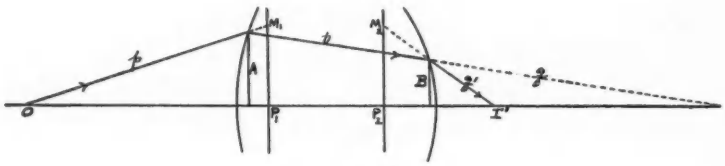


FIG. 9.

For the first surface,

$$\frac{n}{q+t}+\frac{1}{p}=\frac{1}{f_1},$$

from Eq. (1); whence,

$$q+t=nf_1p/(p-f_1).$$

For the second surface,

$$\frac{1}{q'}-\frac{n}{q}=\frac{1}{f_2},$$

from Eq. (2); whence,

$$q=nf_2q'/(f_2-q').$$

Then

$$\frac{A}{B}=\frac{nf_1p/(p-f_1)}{nf_2q'/(f_2-q')}=\frac{f_1p(f_2-q')}{f_2q'(p-f_1)};$$

or

$$B=A\frac{f_2q'(p-f_1)}{f_1p(f_2-q')}.$$

Then,

$$\frac{P_1M_1}{P_2M_2}=\frac{p+\alpha}{q'+\beta}\frac{(f_2-q')f_1}{(p-f_1)f_2}.$$

But for any object and its corresponding image,

$$\frac{\text{size of object}}{\text{size of image}}=\frac{y_1}{y_2}=\frac{p+\alpha}{q'+\beta}.$$

Therefore,

$$\frac{P_1M_1}{P_2M_2}=\frac{y_1}{y_2}\frac{(f_2-q')f_1}{(p-f_1)f_2}.$$

Now, from Fig. 10, a ray passing from a point in y_1 through f_1, the principal focal point of surface S_1, must pass between the surfaces parallel to the axis. It strikes surface S_2 at a point a distance A above the axis and emerges to pass through f_2, the principal focal point of surface S_2, if produced beyond the image y_2. From Fig. 10,

$$\frac{y_1}{p-f_1}=\frac{A}{f_1},\quad A=\frac{f_1y_1}{p-f_1};$$

and

$$\frac{y_2}{A}=\frac{f_2-q'}{f_2},\quad A=\frac{f_2y_2}{f_2-q'}.$$

Then

$$f_1y_1/(p-f_1)=f_2y_2/(f_2-q'),$$

or

$$\frac{f_1y_1}{f_2y_2}\frac{(f_2-q')}{(p-f_1)}=1,$$

and therefore $P_1M_1=P_2M_2$.

On account of this relationship for any two corresponding points in object and image, the principal planes are said to be conjugate. An object point in one plane will be imaged in the other plane in a point such that object and image points are the same distance from the axis. This relationship also accounts for the statement that the principal planes are planes of unit magnification.

VI

From Fig. 11 another important principle can be deduced. Let y_1 be the object, y_2 the final image, and y_x the virtual image formed by the first refraction. The surfaces have radii R_1 and R_2, with centers C_1 and C_2, respectively. Let the

distance from S_1 to either top or bottom of y_1 be p, and let the corresponding distance from S_1 to either top or bottom of y_x be q. Let the distance from S_2 to either top or bottom of y_2 be q', and let the distance between surfaces be t. These assumptions are permissible since the distances from the axis are small compared to the radii and the angles are also small.

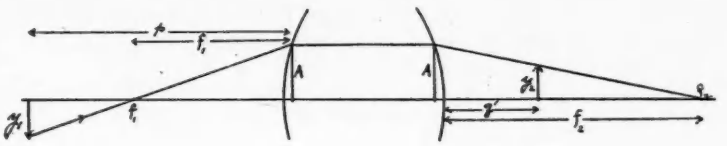


FIG. 10.

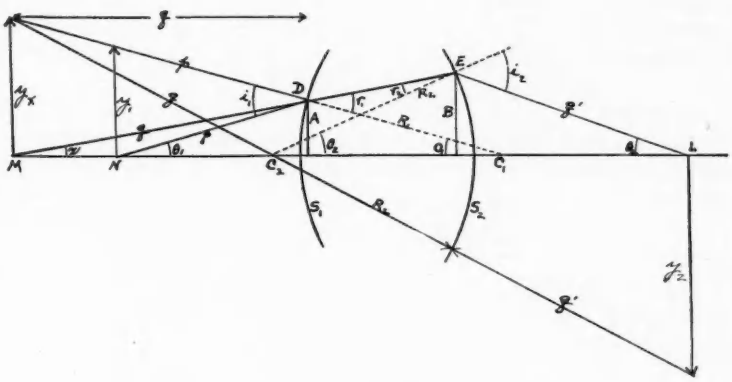


FIG. 11.

From the similar triangles having a common apex at C_1,

$$\frac{y_1}{y_x}=\frac{p+R_1}{q+R_1};$$

moreover,

$$\frac{i_1}{o_1}=\frac{p+R_1}{p},$$

where, in the triangle NDC_1, the sines of small angles have been set equal to the angles. Similarly, from the triangle MDC_1,

$$r_1/o_1=(q+R_1)/q.$$

Then

$$\frac{i_1}{r_1}=\frac{(p+R_1)q}{(q+R_1)p}=n_1,$$

where n_1 is the refractive index obtaining at S_1. Now $\theta_1=A/p$; therefore,

$$y_1=y_x\frac{p+R_1}{q+R_1}=y_x\frac{p}{q}n_1,$$

and

$$y_1\theta_1=y_x(A/q)n_1.$$

From the triangles y_xC_2 and y_2C_2,

$$\frac{y_x}{y_2}=\frac{q+t-R_2}{q'+R_2};$$

from triangle C_2EL,

$$\frac{i_2}{o_2}=\frac{q'+R_2}{q'};$$

and from triangle MC_2E,

$$\frac{o_2}{r_2}=\frac{q+t}{q+t-R_2}.$$

Then

$$\frac{i_2}{r_2}=n_2=\frac{q'+R_2}{q'}\frac{q+t}{q+t-R_2}.$$

Also,

$$y_x=y_2\frac{q+t-R_2}{q'+R_2}=y_2\frac{q+t}{n_2q'}.$$

But $\theta_2=B/q'$, and $B=A(q+t)/q$; then,

$$\theta_2=A(q+t)/qq'.$$

Therefore,

$$y_1\theta_1=y_xn_1\frac{A}{q}=y_2\frac{q+t}{n_1q'}n_1\frac{A}{q}=y_2\theta_2\frac{n_1}{n_2}.$$

Now the absolute index of refraction of the object space,

$$\mu_1=\mu_0/n_1,$$

where μ_0 is the absolute index of refraction of the

medium between S_1 and S_2 and n_1 is the relative index of refraction of the medium referred to that of the object space as unity.

Similarly the absolute index of refraction of the image space

$$\mu_2 = \mu_0 / n_2$$

where n_2 is the relative index of the medium between S_1 and S_2 referred to that of the image space as unity.

Therefore

$$n_1/n_2 = \mu_2/\mu_1.$$

This ratio of absolute indices substituted in the above equation yields the Lagrange-Helmholtz equation,

$$y_1\theta_1\mu_1 = y_2\theta_2\mu_2. \tag{19}$$

It is to be noted that one may speak of initial and final, instead of first and second, refractions if there are several refractions between the initial and final ones. The principle derived here from two refractions applies to multiple refractions.

VII

As an application of the foregoing principles, it is interesting to consider a sphere as a thick lens. Suppose the sphere has a radius R and a refractive index n of 1.5. Then, from Eqs. (1) and (2), f_1 and f_2 are each equal to $2R$. The thickness t is also equal to $2R$. Then

$$\alpha = \frac{f_1 t}{n(f_2+f_1)-t} = R,$$

and

$$\beta = \frac{f_2 t}{n(f_2+f_1)-t} = R.$$

The principal planes are coincident and at the center of the sphere. Moreover,

$$F_1 = F_2 = \frac{nf_1f_2}{n(f_2+f_1)-t} = 1.5R.$$

It is also seen that when $p+\alpha$ has such a value that the image is at infinity, then $1/(q'+\beta)=0$ and $1/(p+\alpha)=1/F_1$; that is, $p=F_1-\alpha=0.5R$. So when the object moves from the surface of the sphere to a point distant $0.5R$ beyond the surface, the virtual image moves between a point distant $2R$ from the surface and infinity. If p is more than $0.5R$ from the surface, a real image is produced on the side of the sphere opposite the object. Evidently the magnification is

$$M = y_2/y_1 = (q'+\beta)/(p+\alpha),$$

which is unity when p and q' are each equal to $2R$ from the surface. The Wollaston-Brewster magnifier known as the Coddington lens is such a lens.

But science also is man's destiny—
Whereby 'tis granted to his working brain,
His industry, his patience and resolve,
To change his old relations with the law
Of space and time; henceforth dependent made
On man's advance in knowledge, and the power
Of using knowledge.—RICHARD HENGIST HORNE

Procedures for the Placement of Transfer Students with Advanced Standing in Physics

BERNARD B. WATSON
University of Pennsylvania, Philadelphia 4, Pennsylvania

WITH the return to the colleges of large numbers of discharged servicemen, departments of physics are faced with the problem of evaluating the courses in physics taken by these men either as civilians before induction into the services or while in uniform in the various Army and Navy training programs. This evaluation does not for the most part involve the assignment of academic credit for work taken, since the determination of the amount of credit to be allowed is normally the function of the registrar or other officer of admission; it does, however, involve the determination of a student's preparation for advanced work in physics and for courses with physics as a prerequisite or of his need for refresher work in physics.

Before the war, transfers between colleges were relatively few in number, and while some of the technical schools administered examinations to transfer students for the purpose of determining their proper classification in physics and the other basic sciences, many colleges accepted a student's transcript at face value. With a larger influx of students presenting credit for physics, more physics departments may be interested in establishing procedures for determining a student's retention of the principles of physics and his possible need for short refresher courses.

Development of procedures for the classification of students presenting college credit in physics was made necessary at the University of Pennsylvania by the establishment in July 1944 of one of the three NARU, or Naval Academic Refresher Units (V-7). This program was set up by the Navy to prepare enlisted personnel of all ranks for entrance to a Naval Reserve Midshipmen's School. To be eligible for assignment to NARU an enlisted man was required to possess, in addition to the usual qualifications for a naval officer candidate, a minimum academic background of two years of college work, including one-year courses in English and mathematics. One year of physics was strongly recommended, but was not required.

Men were sent to each NARU at intervals of four weeks, each class consisting of approximately 70 men. The total number sent to the University of Pennsylvania in the first seven classes, on which this study has been based, was 520. These men came from 47 states and the District of Columbia. They presented academic credits from 321 different colleges and universities. They had been away from college work for periods of time ranging from one to 12 years. The initial function of the staff of the NARU was to interview and examine the men and then to classify them into three groups:

Group I. Men who were able to demonstrate a knowledge of English, mathematics and physics equivalent to that which would be accepted as satisfactory at the termination of the first-year college courses in these subjects; these men were recommended for immediate transfer to one of the Midshipmen's Schools.

Group II. Men who could reasonably be expected to meet the standards set for Group I after eight weeks of refresher instruction in English, mathematics and physics.

Group III. All others; these men were given 16 weeks of instruction, the maximum allowed under the V-7 program.

So far as their training in physics was concerned, Group I consisted, with rare exceptions, of men who had taken formal college courses in physics and who had retained a sufficiently good grasp of physical principles to meet the minimum standards set for this group. Group II was similarly constituted except, perhaps, that the men had not had initially as good a grasp of physics or were further removed from their college work. Group III consisted of men who either lacked college work in physics or no longer had the knowledge of the science needed to pass an examination.

The tests used as the basis for the classification of the men into these three groups were the *Cooperative Physics Tests for College Students.* Five tests—in mechanics, heat, sound, light, and electricity—were administered to each incoming class. The scores were converted into percentiles based on norms established from the results of

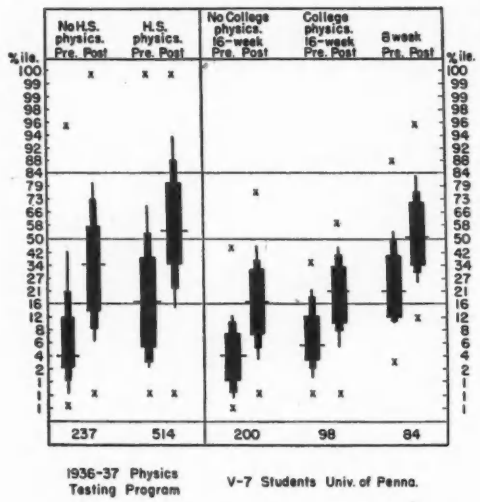


FIG. 1. Results on the mechanics test. Each of the bars represents a homogeneous group of students. The wide portion of each bar represents the range of scores of the middle half of each group. The narrow parts extend to the 16th and 84th percentiles in each group. The lines at the ends extend down to the 10th percentile and up to the 90th percentile. The crosses below the bars represent the lowest scores; and those above, the highest scores in the several groups. The short crossline near the middle of each bar represents the median score of the group. The middle horizontal line in each chart shows the national median for men students, and the other two lines are at the 16th and 84th percentiles of the national distribution.

thousands of similar tests given to college students throughout the nation in connection with the testing program of the American Association of Physics Teachers. The determination of what would constitute a satisfactory performance on these tests presented some difficulty. It was more or less arbitrarily decided that an individual would be considered to have a satisfactory grasp of physical principles on the first-year college level if he ranked in the upper four-fifths of the national group on each of the five tests. Accordingly, a V-7 student was certified in physics for assignment to Group I if he placed in the 20th percentile or above on all five tests. The minimum requirement for assignment to Group II was that the student present credit for a college course in physics and that he place in the 10th percentile or above in mechanics and electricity. An alternative minimum requirement for assignment to Group II was that the student present credit for a college course in physics and that he place in the 10th percentile or above in heat and light, and in either mechanics or electricity. The results of the test on sound were so discordant with those of the other tests that we decided to give little weight to the sound test.

The distribution of men among the three groups is given below. The recommendations indicated were based on the student's performance on the physics tests. The final assignment in each case represented a pooling of the recommendations of the departments of English, mathematics and physics:

Group I. Of 46 men finally assigned to this group, 38 had been certified in physics, and 8 had been recommended to remain for 8 weeks.

Group II. Of 105 men assigned, 85 had been recommended for 8 weeks of physics, and 17 for 16 weeks; 3 were assigned without recommendation on the basis of their college transcripts.

Group III. Of 369 men assigned, there were 98 with credit for one or more courses in college physics, 210 with no college physics, 10 with no record of physics courses (no transcripts received), and 51 who had been recommended to remain for 8 weeks.

For the men assigned to Group I and sent immediately to Midshipmen's School we had no direct way of determining whether our classification procedures had been satisfactory. That our minimum standard for assignment to Group

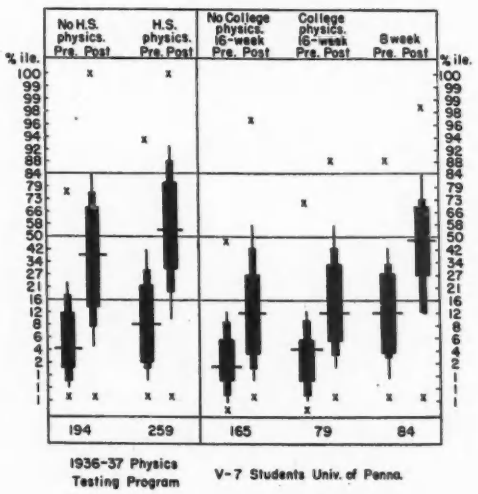


FIG. 2. Results on the test in light.

I was not seriously misplaced, however, is evident from the fact that there were no academic failures in Midshipmen's School among these 46 men. For the other groups we were able to check our selection methods by administering another form of the *Cooperative Physics Test* as the final examination of the physics course. The results of the prestudy and poststudy tests in mechanics, light and electricity are shown in Figs. 1, 2 and 3. For comparison, results obtained in 1936–37 for groups of college students with and without high school work in physics are included.

An inspection of the charts shows that the men selected for the eight-week group had a distribution on the poststudy tests similar to that of the national group which had taken physics in high school; that the 16-week group with credit for college physics was, at the end of the course, almost identical in achievement with the group which had not had a previous college course in physics; and that both 16-week groups were considerably below the national distribution.

These results appear to indicate that we had attained our objective with respect to the eight-week men since at the end of eight weeks these men, as a group, compared favorably with the better of the two groups in the national testing program. It is further evident from the graphs that the mere presence on a student's transcript of credit for a course in physics is in itself no guarantee that the student knows anything about the subject or even that a short refresher course would suffice to bring him to a satisfactory level of competence in elementary physics. The showing of the men in Group III with entrance credit in physics was no better than that of the men without previous college work in physics.

The *Cooperative Physics Test for College Students*, used with the standards found suitable in the V-7 testing program, thus offers a rapid means for classifying the large numbers of former servicemen returning to college with credit for courses in college physics. Some will be ready to go directly into advanced courses; some will require a short refresher course; some should be advised to repeat the general physics course.

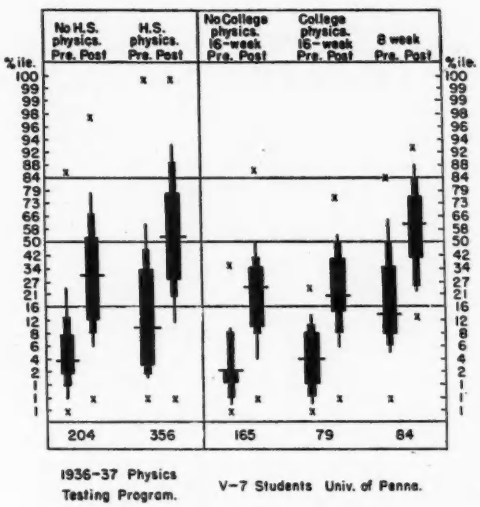


FIG. 3. Results on the test in electricity.

IT is natural to every man to wish for distinction; and the praise of those who can confer honor by their praise, in spite of all false philosophy, is sweet to every human heart; but as eminence can be but the lot of a few, patience of obscurity is a duty which we owe not more to our own happiness than to the quiet of the world at large.—SYDNEY SMITH.

Reproduction of Prints, Drawings and Paintings of Interest in the History of Physics

26. George Stephenson and the First Public Railway

E. C. WATSON
California Institute of Technology, Pasadena 4, California

THE most famous of all the early locomotive builders was GEORGE STEPHENSON (1781–1848). "Although he cannot be described as the inventor of the locomotive, he certainly did more towards its improvement during the crucial period than any other man and, by concurrent construction of suitable railways, did much to establish its practical utility."[1]

Since a number of accounts[2] of STEPHENSON'S life and work are available, one will not be attempted here. Suffice it to say that, having had his interest in steam traction aroused by the success of BLENKINSOP[3] and HEDLEY at Wylam, he in 1814 built a locomotive for the tramway between the Killingworth Colliery, where he had in 1812 been appointed enginewright, and the shipping port nine miles away. This locomotive, which was modeled after that of BLENKINSON, was the first to run by adhesion on iron edge-rails. Between 1814 and 1822, moreover, he made a number of improvements in this engine, and several of the improved types were built and used successfully. In 1823, having succeeded in impressing the advantages of steam traction on EDWARD PEASE, the promoter of a railway from Darlington to Stockton to transport the coal wealth of Durham County to North Sea ports, he was appointed engineer of the railway, with permission to carry out his own plans. The success of the Stockton and Darlington Railway led to his employment in the construction of the Liverpool and Manchester Railway, the first great line and the one which established the railway system as truly successful. In 1823 also, anticipating a demand for engines, he and his son ROBERT, with the backing of PEASE, opened in Newcastle the first locomotive works in the world. "This establishment was the chief seat of practical locomotive development for the ensuing twenty years, and a training ground for locomotive engineers."[4]

PLATE 1. George Stephenson, 1781–1848.

[1] *Handbook of the Science Museum, Land transport. III. Railway locomotives and rolling stock. Part I—Historical review* (H.M. Stationery Office, London, 1931), p. 11.

[2] Samuel Smiles, *Story of the life of George Stephenson* (London, 1857; new ed., 1873); Samuel Smiles, *Lives of the engineers.* Vol. III. *George and Robert Stephenson* (London, 1868; new ed., 1904); C. Matschoss, *Great engineers* (London, 1939), pp. 171–189; R. H. Thurston, *A history of the growth of the steam-engine* (Ithaca, 1931), pp. 183–204; J. S. Jeans, *Jubilee memorial of the railway system. A history of the Stockton and Darlington Railway* (London, 1875), pp. 219–230.

[3] See in this connection the ninth reproduction in this series, *Am. J. Physics* **8**, 46 (1940).

[4] Reference 1. Note that "locomotive engineers," as used in this quotation, refers to builders of locomotives.

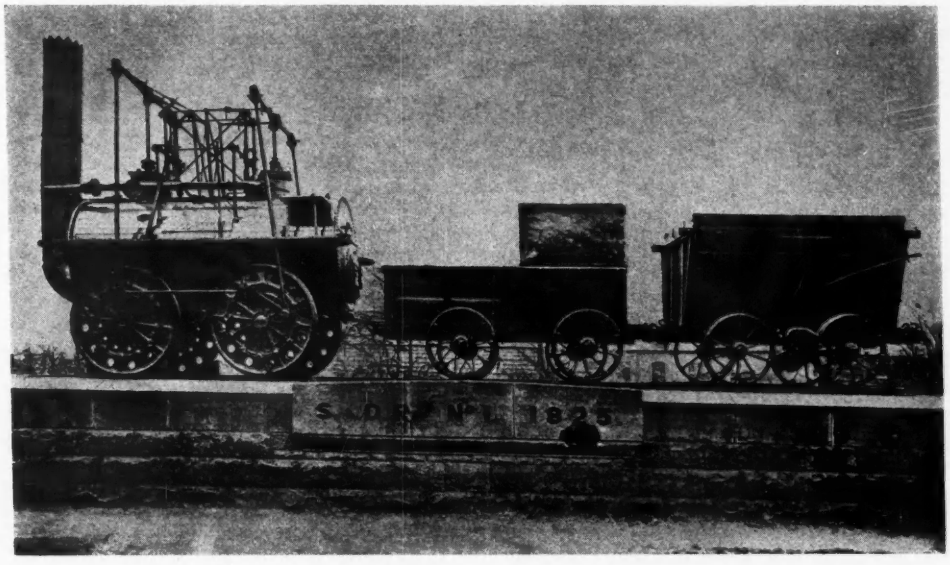


PLATE 2. The first locomotive used on a public railway.

The Stockton and Darlington Railroad was the first public steam railway in the world. At the opening, on September 27, 1825, a train of 34 vehicles, making a gross load of about 80 tons, was drawn by one engine driven by STEPHENSON with a signalman on horseback in advance. The train moved off at the rate of from 10 to 12 mi/hr and attained a speed of 15 mi/hr on favorable

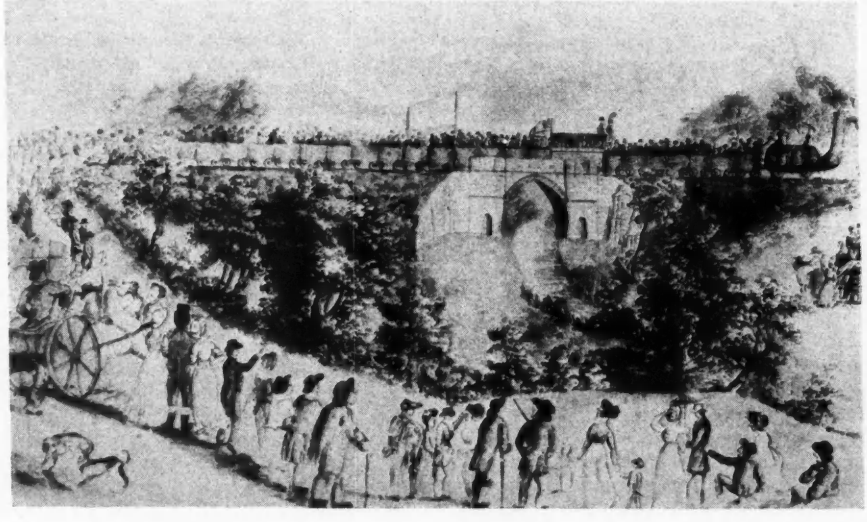


PLATE 3. The inauguration of the first public railway, September 27, 1825.

parts of the line. The locomotive, appropriately called "Locomotion," was built by the STEPHENSONS at Newcastle. It weighed about 7 tons, developed about 10 hp, and hauled a load of from 60 to 70 tons at an average speed of 5 mi/hr.

Plate 1 reproduces a full-length portrait of STEPHENSON by JOHN LUCAS. It is taken from the well-known engraving by T. S. ATKINSON. The section of the Liverpool and Manchester Railroad across Chat Moss, a bog some 12 mi^2 in extent, is shown in the background.

Plate 2 shows the "Locomotion" mounted on a pedestal in front of the Darlington North Road Railway Station, where it was placed with suitable ceremonies in June, 1857.

Plate 3 is reproduced from a drawing by DOBBIN in the Science Museum, London. It portrays the opening of the Stockton and Darlington Railway. As JEANS wrote 50 years later, many curious recollections of the opening ceremony were preserved by those who were present:

> One can scarcely even now meet with a man or woman of advanced years between Auckland and Darlington, both inclusive, who did not "assist" in the opening celebration. It was commonly reported at the time, and has since been handed down as a reminiscence of the event, that the whole of the inhabitants turned out to witness the advent of the "iron horse," save and except two old ladies whose infirmities or prejudices, or both combined, prevented them from rendering so marked a meed of homage to the new era. Great excitement prevailed among the spectators as the engine came in sight. Excitement in many minds took the form of disappointment when it was found that the locomotive was not built after the fashion of a veritable four-footed quadruped, some of the older folks expecting to see the strange phenomenon of an automatical semblance of a horse stalking along on four legs. But everybody admitted that the performance of old "Locomotion" was wonderful in its way, and vigorous cheering greeted its approach.[5]

[5] J. S. Jeans, *Jubilee memorial of the railway system* (London, 1875), pp. 71–72.

MOST of the historians of science have been at pains to efface personality from the individual humans whom we call scientists. They regard this peculiar expunging as part of their duty, a sort of rule of public decency. They are applauded in this by the modern scientists, themselves a prudish lot, constantly afraid that you will discover that they are human. The eulogies that scientists write of each other in the black-leaded obituary pages of their tedious little journals are like country photography, where every wrinkle is touched out. To judge only from these accounts the same scientist keeps dying all the time; I defy you to tell the difference between his incarnations.—DONALD CULROSS PEATTIE, *Green Laurels.*

NOTES AND DISCUSSION

Demonstration Experiment: Magnetized Ring

ERIC M. ROGERS
Princeton University, Princeton, New Jersey

IN discussions of the "molecular theory of magnetism," the idea of a ring that "is magnetized and yet shows no poles" is a valuable one. It illustrates one of the finest uses of a theory: to suggest meaning for an otherwise meaningless statement. So it is important to be able to show that we *can* have a ring that meets this description and reveals its magnetization—as theory suggests it will—when it is cut. Rings that are already cut into semicircular halves are difficult to magnetize uniformly; they show some leakage however well the faces of the gaps are ground.

A simple ring that can be magnetized, tested and then broken can be made from a cheap hacksaw blade, the hole in one end of the blade being used for the central hole in the ring. The end of the blade is softened and cut off; the piece is then snipped or ground to make its periphery concentric with the hole. This ring is then hardened, and magnetized circularly. Tested with iron filings it shows little or no external field. Snapped in two with two pairs of pliers, it reveals strong poles at the new edges. The tests with filings are made in a projection lantern for horizontal slides and show clearly the effect of breaking.

The pliers used should be carefully demagnetized beforehand. The magnetizing of the ring is done by threading thin insulated copper wire several times through the hole, carrying the wire well away from the ring in a large loop after each transit, arranging the loops symmetrically and sending a large current through them for a short time. Rings from thicker blades and better ones that cannot be softened can of course be made by grinding.

Demonstrator Boards for Teaching Electric Circuits

HAYM KRUGLAK
Princeton University, Princeton, New Jersey

AFTER a lecture-demonstration of an electric circuit, the general impression left with the elementary student is often like that following a magician's performance: "It's all done with mirrors." There is a maze (or mess) of wires under, over, in front of and in back of the apparatus; the circuit components are not always arranged to correspond to the symbolic diagrams; there may be a lack of contrast in the background against which the equipment is displayed.

In recent years, especially during the war, considerable use has been made of an improved form of demonstrator board that displays a working circuit with a maximum of visibility and layout simplicity. A schematic diagram of the circuit is drawn on a board; all the parts are mounted near their symbolic representations; the electric connections are made in the back of the board. Terminals, pinjacks, switches and rheostats enable the operator to insert meters and other test equipment, and to change the circuit characteristics.

A typical board is shown in Fig. 1. To operate it, a

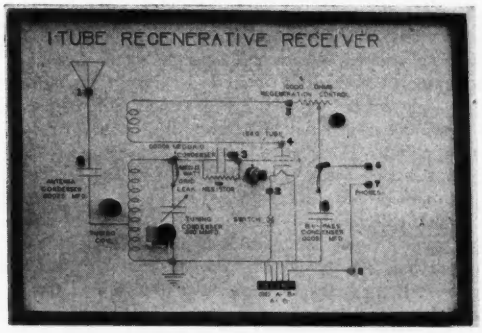


FIG. 1. Radio receiver board.

good, outside antenna is connected to terminal *1*; the terminal *GND* is grounded; a single dry cell is connected to $A+$ and $A-$; a 45-v battery, to $B+$ and $B-$. Earphones are plugged into pinjacks *6* and *7*. The output may also be connected to an audio amplifier with a speaker, in which case the B-battery should be about 90 v. With this board it is possible to demonstrate the tuning of the tank circuit, regeneration, oscillation, detection and amplification. By using a vacuum tube voltmeter it is possible to get the values of grid bias at pinjack *3*; of plate voltage at pinjack *4*; of filament voltage at pinjack *2*; of the battery voltages at the terminal strip. If an instrument such as the Chanalyst is available, it is possible to trace the signal from the antenna to the output and to compare the intensities of the audio- and radiofrequencies at various points.

The close proximity of the actual parts to their symbols helps to bridge the gap between the concrete and the abstract. The orderly arrangement of the circuit elements enables the lecturer to show step by step what happens in each component and its relationship to the entire circuit. Such a board can be carried as a unit to the quiz section or laboratory for additional discussion, or it can be displayed in the museum or hallway.

There are several methods for making a demonstrator board. The difference lies chiefly in the reproduction of

the schematic diagram. Commercial boards are prepared from a stencil with paint brushed on or sprayed on. But the cost of such a method is prohibitive for anything less than mass production. In another method[1] the circuit is drawn to about one half of board size. The drawing is then photographed and enlarged. This enlargement is glued onto a piece of plywood while still wet; when the surface is dry, clear lacquer is applied and the plywood is fitted into a frame. The disadvantage of this method is that making a 2×3-ft enlargement may not be convenient or feasible. The advantage, of course, is that several copies can be made.

A third method obviously is to make a full-size drawing and trace it, or draw it directly, on some tracing material. A black-on-white print is made by the usual drafting-room procedure, glued onto plywood or stiff cardboard, and set into a frame. This method is also useful when duplicate copies are desired. Where only a single board is required, the simplest and quickest method is to draw the circuit on heavy light-colored cardboard with pencil. Colored pencils or crayons are especially effective in emphasizing component circuits. If a change is desired, erasing and redrawing of a section becomes a simple matter. The cardboard is then nailed onto a frame (Fig. 2).

After the circuit is drawn, the surface should be coated with some preservative such as clear lacquer, preferably by spraying. The parts are then mounted by means of screws, bolts or brackets. Care must be taken to arrange the physical components so as not to obscure any section of the drawing, the labeling, or any other part. Permanent connections are soldered in the back of the board. Where a section of the circuit is to be shorted out, or where a meter, oscilloscope or other test equipment is to be inserted, pinjacks or lug terminals should be provided in front of the board. The frame should be provided with carrying handles and mounting feet.

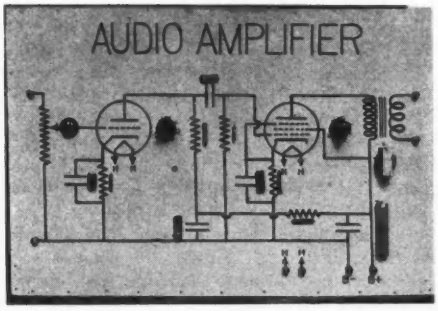


FIG. 2. Audio amplifier board.

The cost of a demonstrator board need not be high. Students with amateur radio experience are glad to do this work for regular assistant pay. Old radio sets are good sources of parts; the junk drawers and shelves will provide many components. If an expensive piece of apparatus is needed for other purposes as well, it can be mounted temporarily by means of screws or clamps.

The following list is merely suggestive of the possible demonstrations with the boards:

Ohm's law	Hartley oscillator
Power[2]	Audio amplifier
Resonance	One-tube receiver
Kirchhoff's laws	Bridge rectifier
Wheatstone bridge	Diode and triode characteristics
Potentiometer	Photoelectric circuit
Power supply	Nonlinear resistances

Modulation and detection

In a course in applied electronics the superheterodyne six-tube dynamic demonstrator board (RCA) is extremely useful.

The method of demonstrator boards is also readily applicable to lenses, optical instruments and other devices in geometrical optics.

The author wishes to express his appreciation to Drs. R. Schlegel and J. Walter for many helpful suggestions, and to Mr. H. Waage for invaluable technical assistance.

[1] D. B., "New laboratory techniques expedite training in electronics," *Electronics*, pp. 90–93 (June 1943).
[2] J. T. Peters, "Electric wiring and apparatus board," *Am. J. Physics (Am. Phys. T.)* **7**, 137 (1939).

A Larger Gyroscope

GEORGE P. UNSELD
West High School, Salt Lake City, Utah

HAROLD K. SCHILLING'S recent article [1] suggests that we describe a large gyroscope that has afforded much interest and even astonishment. We fitted the flywheel of a small engine to a piece of 2-in. pipe by using the ball bearings from the axle of an old car. A hole was

FIG. 1. Gyroscope made from an engine flywheel.

drilled in the pipe 6 in. from the center of the wheel. One end of a support rod was ground until it fitted the hole very loosely. When the rod was firmly attached to a table, and its upper end inserted in the hole in the 2-in. pipe, there was enough play to allow the axle to drop until the wheel touched the rod.

With both surprise and delight we found that sufficient energy can be imparted to the wheel by a single vigorous turn of the hand to cause it to precess around the vertical rod, the axle remaining horizontal, or even with the free end pointing upward at an angle. We have found it advisable to start the precession by giving a slight push in the proper direction. The wheel weighs 40 lb, but it is not hard for one person to perform the experiment alone by resting the free end of the axle in the hollow of his left arm. Once, before we had the bearing properly secured, the wheel came loose, fell, and broke through one of the boards of the floor.

In our study of the gyroscope we find the "straightedge rule" useful. As this simple statement is not so commonly known, we give it for the benefit of those who have occasion to discuss precession. *Any force applied to a rotating gyroscope can be thought of as caused by the pressing of a straightedge against the axle. The direction in which precession occurs is the same as that in which the straightedge would roll along the axle.*

[1] H. K. Schilling, "Demonstrations with a large, low speed gyroscope," *Am. J. Physics* **14**, 116 (1946).

The Falling Chimney

ARTHUR TABER JONES
Smith College, Northampton, Massachusetts

ABOUT ten years ago Richard Sutton[1] discussed briefly the breaking of a chimney while it is falling. His reasoning depended on the position of the center of percussion, and he described two simple and rather surprising demonstrations that are related to the problem. Treatments of the stresses in the chimney have been given by Reynolds[2] and by Bundy,[3] and the latter reproduced photographs of four falling chimneys. The following simple reasoning is sufficient to give an answer to the question as to which way the chimney will buckle when it falls.

Let the chimney be replaced (Fig. 1) by two stiff and weightless rods ab and bc, hinged at a and b, and two equal loads, one at b and one at c. Let the rods lie at first along a straight line which is inclined to the vertical, as in the figure.

Since the two weights are equal, their components F_b and F_c perpendicular to the direction ac are also equal. If $ab=bc=l$, the torques which these components exert about a are $L_b=lF_b$ and $L_c=2lF_c$. Since $F_c=F_b$, it follows that

$$L_c=2L_b. \tag{1}$$

If the mass of each load is m, the moments of inertia about a are $I_b=ml^2$ and $I_c=4ml^2$, so that

$$I_c=4I_b. \tag{2}$$

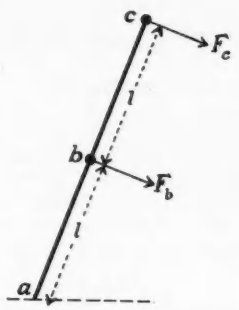


Fig. 1. Simplified chimney.

If α stands for angular accelerations, we have in general $L=I\alpha$, and therefore, from Eqs. (1) and (2),

$$\alpha_b=2\alpha_c. \tag{3}$$

It follows that the load at b acquires angular velocity more rapidly than that at c, and the chimney buckles in the direction that the photographs[3] show.

[1] Richard M. Sutton, *Science* **84**, 246 (1936); *Demonstration experiments in physics* (McGraw-Hill, 1938), p. 89.
[2] J. B. Reynolds, *Science* **87**, 186 (1938).
[3] F. B. Bundy, *J. App. Physics* **11**, 112 (1940).

More on the Dynamics of a Roll of Tape

LEONARD T. POCKMAN
Heintz & Kaufman, Ltd., South San Francisco, California

IN his article on "Dynamics of a roll of tape," Ira M. Freeman[1] develops a mathematical analysis of the paradox that I proposed some time ago.[2] Further physical insight into the nature of the paradox may be gained by observing that the rolling motion is not maintained during the entire process. The last fraction of a turn does not form a rigid wheel and is free to whip or slap the plane and suddenly dissipate the accumulated kinetic energy.

During a summer in the mountains I constructed a model to illustrate this point. It was formed by weaving about a pound of decapitated sixpenny nails at right angles to a long narrow strip of mosquito netting. The inclined plane was formed by a large inverted dishpan. With a steep incline, near vertical or vertical, the tape unrolled quietly until the last nail or two struck the pan with a resounding slap. The degree of concentration of the kinetic energy might have been determined by placing a piece of paper covered with carbon paper at the bottom of the incline. This crude but nice experimental demonstration was developed to illustrate the theoretical explanation which I had previously used with my classes.

[1] *Am. J. Physics* **14**, 124 (1946).
[2] "Nonconservation of energy—a paradox," *Am. J. Physics* **9**, 51 (1941).

RECENT MEETINGS

Oregon Section

THE Oregon Section of the American Association of Physics Teachers met at the Public Library Building, Portland, Oregon on February 16, 1946 in conjunction with the annual meeting of the Oregon Academy of Science. Following a trip through the plant of the Aluminum Company of Vancouver, Washington, the following program was presented.

Microwave antenna measurements. T. W. LASHOF, *Reed College.*
Application of optical theory to microwave antennas. J. J. BRADY, *Oregon State College.*
Teaching radar to naval officers. H. R. VINYARD, *Oregon State College.*
Radar countermeasures. E. A. YUNKER, *Oregon State College.*

W. R. VARNER, *Secretary*

Indiana Section

THE Indiana Section of the American Association of Physics Teachers met at Purdue University on June 1, 1946. Approximately 48 members and guests attended. The program was as follows:

Slot antennas. J. B. HERSHMAN, *Valparaiso Technical Institute.*
The impending breakdown in research library facilities. DUANE ROLLER, *Wabash College.*
Coordinated science curriculum. R. W. LEFLER, *Purdue University.*
Color mixing, a demonstration. CHARLES H. SKINNER, *Indiana University.*
Report on the January meeting of the Association. I. WALERSTEIN, *Purdue University.*
Yo-yos. A. R. THOMAS, *Valparaiso University.*
Microwave radar. R. O. HAXBY, *Purdue University.*

The meeting concluded with an inspection of the research laboratories and a dinner in the Purdue Memorial Union.

K. LARK-HOROVITZ, *Chairman*

Southern California Section

THE annual spring meeting of the Southern California Section of the American Association of Physics Teachers was held at Los Angeles City College, on March 16, 1946. An invited paper on radar was read by H. A. Kirkpatrick, Naval Ordnance Laboratory, Inyokern, California, formerly at the Radiation Laboratory, Massachusetts Institute of Technology. The following contributed papers were presented.

A graphical comparison of the Fahrenheit and centigrade temperature scales. J. W. ELLIS, *University of California at Los Angeles.*
The flowing spring fed through sand. L. E. DODD, *University of California at Los Angeles.*
A pendulum. D. B. PHELEY, *Los Angeles City College.*
Parallel operation of cold-cathode glow-discharge tubes in voltage-regulator circuits. V. L. BOLLMAN, *Occidental College.*
Progressive release of potential energy: two demonstrations. L. E. DODD, *University of California at Los Angeles.*
Maintaining student interest in physics. D. L. SOLTAU, *University of Redlands.*
An electronics teaching aid. R. W. LEONARD, *University of California at Los Angeles.*

V. L. Bollman reported that plans are complete for the Second Annual Competitive High School Physics Test to be held on June 1, 1946 and that scholarships having a value of more than $1500 will be available to winners in the examination.

The following officers were elected for 1946: *President*, V. L. Bollman, Occidental College; *Vice President for colleges*, L. E. Dodd, University of California at Los Angeles; *Vice President for junior colleges*, D. B. Pheley, Los Angeles City College; *Vice President for high schools*, G. G. Gordon, South Gate High School; *Secretary-Treasurer*, R. W. McHenry, Santa Monica High School; *members of Executive Committee*, R. E. Worley, University of California at Los Angeles; D. L. Soltau, University of Redlands.

VERNON L. BOLLMAN, *Secretary-Treasurer*

Colloquium for College Physicists at University of Iowa

THE successful University of Iowa colloquiums for college physicists, held annually before the war, were resumed with the meetings held on June 13 to 15, 1946. Some 108 physicists and graduate students from 55 institutions in 16 states were in attendance. The program follows:

German war physics. GREGORY WANNIER, *University of Iowa.*
Radar counter-measures. J. J. LIVINGOOD, *Collins Radio Company.*
Handling of units in physics problems. J. H. HARTY, *Southeast Missouri State Teachers College;* W. H. KADESCH, *Iowa State Teachers College.*
Demonstrations and exhibits. F. W. COOKE, *State Teachers College, St. Cloud, Minn.;* C. A. CULVER, *Carleton College;* W. H. ELLER, *Illinois State Teachers College;* Z. V. HARVALIK, *State Teachers College, Duluth, Minn.;* G. W. HEITKAMP, *Loras College;* W. J. HOOPER, *Principia College;* R. R. PALMER, *Beloit College;* C. R. SMITH, *Aurora College;* L. W. TAYLOR, *Oberlin College;* E. R. WIGHTMAN, *U. S. Signal Corps;* J. W. WOODROW, *Iowa State College.*
Micrometeorology and atmospheric ultrasonics. H. K. SCHILLING, *Pennsylvania State College.*
Microwave radar. L. A. TURNER, *University of Iowa.*
Proximity Fuzes. R. D. HUNTOON, *National Bureau of Standards.*
Influence of the increasing interest in general education on the teaching of college physics. Round table discussion. RUSSELL COOPER, *Assistant Dean, College of Liberal Arts, University of Minnesota;* DUANE ROLLER, *Wabash College;* L. W. TAYLOR, *Oberlin College;* C. N. WALL, *University of Minnesota.*
High spots in published discussions of physics teaching during the war years. Round table discussion. W. P. GILBERT, *Lawrence College;* W. NOLL, *Berea College;* ARTHUR ROUSE, *St. Louis University;* H. Q. FULLER, *Albion College.*
Problems in publication of the periodical literature of physics. DUANE ROLLER, *Wabash College.*

Other features of the program were an exhibit of recent physics motion pictures, discussions of legislation of interest to physicists and of the significance of the various organizations of atomic scientists, several luncheon meetings, and a reception for members and guests at the residence of Professor G. W. Stewart.

DIGEST OF PERIODICAL LITERATURE

Melde's Experiment

A 2-m length of No. 36 S.W.G. cotton-covered copper wire is stretched horizontally between a fixed support and the bob of a pendulum. The pendulum consists of a 50-cm piece of the same wire, wrapped around a peg at the upper end and with a 100-gm bob. The pendulum support is mounted on a wheeled carriage. By moving the carriage back or forth, the force in the horizontal wire can be very accurately adjusted, and it can be measured in terms of the length and displacement of the pendulum. Alternating current of about 0.2 amp from a 3-v transformer is passed through the horizontal wire. An Alnico bar magnet is held vertically with its lower pole about 1 cm above the wire and some 10 cm from one end. The wire then vibrates, and the vibrations can be made quite strong by adjusting the force in the wire.

When good standing waves have been produced, with a rather weak magnetic field, a second magnet is moved along the wire. The amplitude of the vibrations changes in a way that can be used to show that the motions of the wire at two consecutive antinodes are always in opposite directions. E. ARUJA, *Sch. Sci. Rev.* **26**, 166–170 (1945).

Science and Education

The present age is scientific in the sense that the Middle Ages were religious. The man in the street's conception of science and scientific method may be as distorted and uncritical as the medieval villein's conception of religion, but religion was then and science is now, as it were, the language of the multitude. If the modern generation is to be educated it will have to be mainly through the medium of science.

It is not so much that a knowledge of the facts of physical science are essential for an understanding of contemporary problems as that the absence of training in any one of the major branches of knowledge leads to a departmental attitude of mind. The bad temper which the theologian attributes to original sin, the chemist may attribute to calcium deficiency, and each be unaware that his explanation is the consequence of his specialized study. It is not to decry science to remember its departmentality. Every subject is an artificial abstraction from the unity of knowledge, and it is only by an effort of thought that we can distinguish between even the perceptive and the appreciative elements in experience. No single subject can therefore form an adequate basis for a philosophy or be in itself a satisfactory guide to conduct.

The scientist rightly asserts that there is in every international problem a factual element about which agreement is possible if the problem is approached scientifically. There is also, however, in every problem an element of value which is not susceptible to scientific treatment and about which differences of opinion are inevitable.

The real danger of specialization is not so much an ignorance of other branches of knowledge as a failure to be conscious of this ignorance. Science teaching is of little value unless it forms the basis of a liberal education, and it can form such a basis. Under present conditions the choice is very largely between a liberal education on a scientific basis or an education which, though scientific, is not liberal. S. R. WOODS, *Sch. Sci. Rev.* **25**, 138–142 (1944).

Science and Human Beliefs

Rational behavior depends upon belief, and it is important to consider the effect that the teaching of science is having on the beliefs, and so upon the behavior, of the common people. It is hard now to realize the domination of thought by the churches 50 years ago. For instance, the literal truth of the Bible account of the creation was believed then, as it is now the orthodox belief of many Christian churches. Science is today uprooting faith without planting anything more suitable in its place, though science itself is not devoid of inspiration. Yet the urge to seek truth is what leads science forward. Those who are responsible for the examination of religious beliefs are not always sensitive to this urge. But so long as there are churches there will be creeds; and so long as there are creeds there will be obstruction. Any religious system which so binds itself by formal creeds that it cannot embrace newly discovered truth stands self-condemned. Some of the churches have magnified debatable and secondary points into essential articles of belief, though there are many within the churches as well as without who regret this false emphasis.

Yet such motes in ecclesiastical eyes must not be obscured by the beams in the eye of the science teacher. He spends much of his thought on material things. His analysis of the physical world may lead him to believe that he has explained its syntheses. He seldom remembers that mind is the most direct thing in our experience, and that there is a difference between animal behavior and human endeavor. He is apt to think of man's religious sense as a vestige, when it may rather be a rudiment. He often implies that because what he teaches is the truth, it is also the whole truth. Beauty and goodness, though overshadowed of late by scientific achievement, are none the less real. Beauty is not unknown in science, but goodness is seldom mentioned. If the science teacher feels that the churches are failing in their duty or are losing their appeal, he should do what he can to revitalize them. Unless men hold a faith in things transcendental, they rely on sectional interests and partial loyalties, which inevitably lead to bitterness and strife. C. L. BRYANT, *Sch. Sci. Rev.* **26**, 261–268 (1945).

Remagnetizing Permanent Magnets

A coil of several hundred turns of copper wire, say No. 18 or 20, is connected in series with a piece of fuse wire and a switch to the 115-v a.c. line. The magnet to be remag-

netized is placed within the coil. When the switch is closed the fuse wire blows immediately, but the magnet is found to be remagnetized. The method will work with steel or with such alloys as Alnico. H. F. BOULIND, *Sch. Sci. Rev.* **26**, 360 (1945); L. C. DAVISSON, *Sch. Sci. and Math.* **46**, 34 (1946).

Demonstration of Liquid Pressure

A bent glass tube that may be dipped beneath the surface of water in a jar and an open water manometer are jointed by a **T** to a tube leading to a source of compressed air and having a short side tube equipped with a stopcock. Initially the stopcock is open and air escapes from it, so that the manometer reads zero. The stopcock is then slowly closed until air just begins to bubble from the end of the tube in the water. The manometer then indicates the pressure of the water at the depth of the open end of the latter tube. H. W. LE SOURD, *Sch. Sci. and Math.* **46**, 304 (1946).

Composition of Wave Forms

On a track that may be tilted runs a car carrying a strip of paper. A string from one end of the car runs over pulleys at the ends of the track and over an idler pulley at one side for adjusting the tension. The upper pulley is on a shaft that carries a vane in a dashpot containing heavy oil, by means of which the speed of the car may be made very constant.

The vibrating system consists of a flat spring strip with adjustable loads. One end of the strip is held fixed, and the other carries a fine brush, the tip of which is just in contact with the paper on the car. Transverse vibration of this strip results in a sine wave on the moving paper. An additional wave is produced by slight torsional vibration of the strip, governed by weights on a rod clamped to the strip at right angles to it. Frequency ratios, amplitudes and relative phases of the two modes of vibration can be varied through rather wide limits. A. D. BULMAN, *Sch. Sci. Rev.* **27**, 202–206 (1946).

Demonstration of Thermal Diffusion

Thermal diffusion, one of the methods used in separating U^{235} for use in the atomic bomb, is based on the principle that a temperature gradient in a mixture of two gases gives rise to a concentration gradient, the lighter constituent being concentrated in the hotter portion of the containing vessel. The process can be improved by combining it with a thermal convection process.

The present apparatus consists essentially of three concentric glass tubes of diameters 1.2 cm, 3.5 cm, and 5.0 cm. Within the inner tube is a heater of Nichrome wire. Tap water is circulated between the middle and outer tubes. The gas space is between the inner and middle tubes, the temperatures of these tube surfaces being about 25° and 150°C, respectively. The tubes are mounted vertically, and the middle tube is connected to two 3-l flasks, one at the bottom and the other at the top.

The separation is made visible by using an approximately equimolecular mixture of bromine and air. Within 10 hrs the separation is quite apparent, and within a few days most of the bromine originally in the upper flask has been transferred to the lower. If the heater is then turned off, the bromine concentration becomes uniform in about a day. W. M. SPICER, *J. Chem. Ed.* **22**, 593 (1945).

Science and General Education

One important objective of science teaching is to give a practical understanding of scientific method, sufficient to be applicable to the problems that a citizen must face in his individual and social life. The way to learn scientific method is to practice it. The business of finding a problem in a situation, of formulating and solving it and of checking the results can be appreciated only by those who have carried it out.

The problem set must be interesting, significant and real—to the student, not the teacher. Faced with an unreal problem, a student naturally turns it into the real one of finding out what answer the teacher wants him to get. Many technics formerly considered indispensable for a scientific education are later used only by specialists and should be abandoned in general education. The teacher must be concerned with social problems, not as a propagandist but as one who examines and discusses the relevant facts in a situation.

The development of science has given us the means for a fruitful and secure life. The obstacles to that life lie in social forms. Science and education are still powerful weapons for the defense of democracy. J. D. BERNAL, *Sch. Sci. Rev.* **27**, 150–158 (1946).

THERE is a certain compensation for ignorance; that one is led to think at first hand, guided but not dragged along by men who know.—CANON LYTTELTON to J. J. THOMSON.